HUMAN SCIENCES RESEARCH COUNCIL

TIMSS 2003

LEARNING SUPPORT MATERIAL FOR MATHEMATICS BASED ON ERROR ANALYSIS OF TIMSS 2003 ITEMS

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Learning Support Material for Mathematics based on error analysis of TIMSS 2003 items

Introduction

The Trends in Mathematics and Science Study (TIMSS) is a large-scale study conducted by the IEA (International Association for the Evaluation of Educational Achievement). The TIMSS study tested learners from a large number of countries over extensive content domains in Mathematics and Science. The study is conducted (repeated) every four years. In 1995 a study called the Third International Mathematics and Science Study was conducted. In 1999 a repeat of the study was conducted and it was called TIMSS-R and in 2002/2003 the name was revised and it was called Trends in International Mathematics and Science Study (TIMMS). The South African results for Mathematics and Science achievement for TIMSS (1995) and TIMSS-R (1999) were very poor in relation to that of the other participating countries. South Africa scored the lowest amongst all countries that participated in both mathematics and science for both studies. Many questions have been raised about the reason for this but nothing much has changed that might have contributed to an improvement in the achievement in these two subjects.

To be able to bring about change in mathematics achievement in South African schools, TIMSS 2003 study should not only be seen as a tool for constructing reporting scales but as a rich source of information constituted in the data that should be explored to improve learning and teaching in mathematics. Data on individual items constitute a rich source of information, not only on the dimension of right/wrong answers provided but also on the diagnostic aspects of which right or wrong responses students actually gave. General trends of what learners can and cannot do could be of value in itself and some of the specific errors identified might be of practical use in teaching. If educators know the proportion of learners making a given error they will know if they are likely to meet the error in their teaching process and how often they can expect to meet it.

This document is developed as learning support material to help practitioners to identify the type of mistakes learners made by analyzing the answers of learners given to items in the TIMSS 2003 study. By analyzing the common errors made by grade 8 learners in the answers given to mathematics questions in the TIMSS 2003 study and by identifying misconceptions that may be revealed by these errors, the learning support material will attempt to supply information that can inform the teaching process. The information may help educators to dismantle misconceptions and may result in the prevention of learning problems in mathematics. It should assist teachers to integrate assessment in the instruction process and should be used to drive instruction in the classroom.

Because this learning support material forms part of a larger TIMSS study the test design and framework used in the larger study is discussed briefly in the next session to give an indication on how instruments were developed, implemented and scored and how the data were analysed.

TIMSS Conceptual Framework for assessment instruments

The central aim of TIMSS is to measure achievement in Mathematics and Science in order to learn more about the nature, content and context in which learning occur. The assessment frameworks of the TIMSS studies focused on the curriculum as a broad explanatory factor underlying student achievement. It envisaged three "levels" of curriculum, namely what society would like to see taught (the intended curriculum), what is actually taught (the implemented curriculum), and what the learners learn (the attained curriculum). Factors from the educational environment that influence educational decisions are investigated from the perspective of these three curriculum levels.

Instruments and tests

TIMSS use written tests of mathematics and science to measure students' achievement and a series of questionnaires (Curriculum, School, Student, and Science and Mathematics Teacher Questionnaires), that focus on context for students' learning in these subjects to gather information about the context in which learning took place.

The TIMSS 2003 research collected three types of data, namely

- Data on learner achievement,
- Data on the curriculum, and
- Contextual data from principals, teachers and learners

Achievement instruments

The mathematics and science tests were developed internationally in a collaborative manner. Two different types of questions (multiple-choice questions and constructed-response questions) were included in the pool of TIMSS questions. Although items focus on a particular content element it also assume knowledge or skills from other content areas.

To achieve a valid assessment of the two subjects a substantial number of assessment questions were needed. To accommodate the large number of questions required in the limited testing time available the questions were divided among a set of 12 learner booklets. Each learner completed only one of the 12 student booklets in two sessions of 45 minutes. The 12 booklets were rotated amongst the learners in a class so that approximately equal proportions of leareners in the class responded to each booklet. The questions were assigned in such a way to the booklets that the combined responses of individual learners provide a comprehensive picture of the achievement of the entire learner population.

After each TIMSS assessment cycle, some items are released for public use while the others are kept secure to measure trends over time. Replacement items that closely match the content of the original items are developed. Comparable scores for students that took the test in different years are obtained by making use of Item Response Theory (IRT).

Mathematics Framework:

The mathematics assessment framework for TIMSS 2003 consists of two organizing dimensions, a content dimension (the specific subject matter covered) and a cognitive dimension (the expected behaviors of students as they engage with subject content). Each dimension has several domains and each domain consists of several topic areas. The two dimensions and their domains are the foundation of the assessment.

The two dimensions and their domains divided in the different topic areas are outlined below.

Mathematics Content Domains

a) Numbers:
- Whole numbers
- Fractions
- Integers
- Ratio, proportion and percent

b) Algebra:
- Patterns
- Algebraic expressions
- Equations and formulas
- Relationships

c) Measurement:
- Attributes and units
- Tools, techniques and formulas

d) Geometry:
- Lines and angles
- Two and three-dimensional shapes
- Congruence and similarity
- Location and spatial relationships
- Symmetry and transformations

e) Data
- Data collection and organization
- Data representation
- Data interpretation
- Uncertainty and probability

Cognitive Domains:

a) Knowing facts and procedures:
- Recall
- Recognize/Identify
- Compute
- Use tools

b) Using Concepts:
- Know
- Classify
- Represent
- Formulate

- Distinguish

c) Solving Routine Problems:
   - Select
   - Model
   - Interpret
   - Apply
   - Verify/check

d) Reasoning:
   - Hypothesize/Conjecture/Predict
   - Analyze
   - Evaluate
   - Generalize
   - Connect
   - Synthesize/Integrate
   - Solve Non Routine Problems
   - Justify/prove

The achievement tests contain two types of questions. The first type of questions requires learners to select appropriate responses (multiple-choice questions) and the second type requests learners to solve problems and answer questions (constructed-response questions). Each constructed-response question has its own scoring guide, developed to provide data about students' achievement as well as diagnostic information about misconceptions and common errors. The scoring uses two-digit codes with scoring guides specific to each item. The first digit designates the correctness of the response. The second digit, combined with the first digit, represents a diagnostic code used to identify specific approaches, strategies, or common errors and misconceptions.

The number of learners that were allocated specific diagnostic codes for the free-response questions and the number of learners that selected different options for the multiple choice items will be used to analysed the responses to the released items in an attempt to clarify learners' performance. These errors will be analysed to determine if they reveal any misconceptions amongst learners in mathematics. By interpreting the patterns of errors, misconceptions and predominant solution methods of learners we will try to provide some insight into learners ways of thinking and why they made certain errors. The structure of the released multiple choice items is illustrated in Figure 2. The item is shown on the left hand side while the Content Domain, Main Topic, Cognitive Domain and Key (the correct answer) is shown on the right hand side

Figure 2

M032609

Which of these is closest to $11^2 + 9^2$?

(A) $20 + 20$
(B) $20 + 80$
(C) $120 + 20$
(D) $120 + 80$

Content Domain
Number

Main Topic
Whole numbers

Cognitive Domain
Knowing Facts and Procedures

Key
D

The items will be discussed under the headings::

- **Responses of learners**: This section will indicate the percentage of learners that selected the different options in the multiple choice items or the percentage of learners that gave a specific response to an open ended question. The correct option for the multiple choice items will also be indicated under this heading.
- **How to solve the problem**: The way in which the problem can be solved is discussed under this heading. It is important to note that this is only one example of many ways in which the problem may be solved.
- **Errors of learners**: Data on the dimension of which right or wrong responses students actually gave is discussed. General trends of what learners can and cannot do as well as some of the specific errors are identified.
- **Link to RNCS**: Under this heading each question has been attached to a Learning Outcome (LO) and an Assessment Standard from the Revised National Curriculum Statement (RNCS). Reference with regard to the grade/s in which learners should achieve the Assessment Standard and the page number on which it appear in the RNCS are also given.

M032609

Which of these is closest to $11^2 + 9^2$?

(A) 20 + 20
(B) 20 + 80
(C) 120 + 20
(D) 120 + 80

Content Domain
Number

Main Topic
Whole numbers

Cognitive Domain
Knowing Facts and Procedures

Key
D

1. **Responses of learners:**
   A: 47.1%, B: 12.3% C: 12,9% D: 16.0%
   Attempt made but not correct (double response for MC items) 0.7%
   Item omitted 6.3% Item not reached 4.6%

   D is the correct answer

2. **How to solve the problem**
   This item assesses knowledge on whole numbers and exponents and involves three steps.
   Step 1: Work out the square for 11 i.e. 11x11 =121
   Step 2: Work out the square for 9, i.e. 9 x 9 = 81
   Step 3: Round the answers off to the nearest ten to be able to compare it with the options, i.e. 120 + 80.

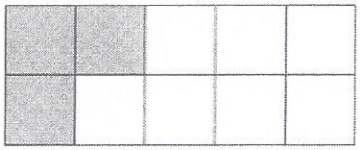
3. **Errors of learners**
   The most frequent error made by learners was selecting option A as the correct answer with 47.1% (nearly three times as many as those selecting the correct answer) of the learners selecting this option. Learners that had selected option A did not work out the squares for the two numbers but simply multiply the approximate value of 11 and the approximate value of 9 with two. This error indicates that these learners have not grasp the concept of exponents.

4. **Link to RNCS**
   Learning outcome 1: Grade 7 learners must be able to *estimate* and work with *exponents, viz squares* (RNCS page 70).

M012001

In the figure, how many MORE small squares need to be shaded so that $\frac{4}{5}$ of the small squares are shaded?

Ⓐ 5
Ⓑ 4
Ⓒ 3
Ⓓ 2
Ⓔ 1



Content Domain
Number

Main Topic
Fractions and decimals

Cognitive Domain
Using Concepts

Key
A

1. **Responses of learners:**
   A: 12.9%, B: 8.4% C: 37.9% D: 7.8% E: 17.7%
   Attempt made but not correct (double response for MC items) 2.2%
   Item omitted 7.0% Item not reached 6.1%

   A is the correct answer

2. **How to solve the problem**
   This item assesses learners' knowledge about fractions and required from them to determine the number of additional squares to be shaded so that $\frac{4}{5}$ of the squares are shaded.

   There are three steps.

   Step 1: In the rectangle of 10 squares, $\frac{4}{5}$ of the squares would be 8
   Step 2: No of squares shaded are 3.
   Step 3: Number of squares still to be shaded is 8 – 3 = 5.

3. **Errors of learners**
   The most frequent error made by learners was selecting option C as the correct answer with 37.9% of the learners selecting this option These learners did not read the question properly, but simply selected the option that represents the number of squares originally shaded, i.e. 3 squares.

4. **Link to RNCS**
   Learning outcome 1: The RNCS requires learners from grade 4 to be able to represent common fractions in diagrammatic format (RNCS p 43) and learners from grade 5 to 7 to calculate by using operations appropriate to solve problems using common fractions with denominations which are multiples of each other (RNCS p 43 and p 70).

M022191

Two-thirds of the people present at the beginning of a meeting are men. Nobody leaves but 10 more men and 10 more women arrive at the meeting. Which of the following statements is true?

(A) There would then be more men than women at the meeting.

(B) There would then be the same number of men as there are women at the meeting.

(C) There would then be more women than men at the meeting.

(D) From the information given, you cannot tell whether there would be more women or men.

Content Domain
Number

Main Topic
Fractions and decimals

Cognitive Domain
Reasoning

Key
A

1. **Responses of learners:**
   A: 18.3%, B: 27.5% C: 11.1% D: 18.7%
   Attempt made but not correct (double response for MC items) 1.6%
   Item omitted 7.2% Item not reached 15.5%

   A is the correct answer

2. **How to solve the problem**
   This item requires from learners to compare the number of men/women at a meeting. It involves two steps.

   Step 1: Number of men originally at the meeting ($\frac{2}{3}$ is men) is more than the number of women originally at the meeting.

   Step 2: Number of men originally at the meeting plus 10 more men are still more than the number of women originally at the meeting plus 10 women

3. **Errors of learners**
   The most frequent error made by learners was selecting option B as the correct answer with 27.5% of the learners selecting this option. To arrive at option B as an answer, learners look only at the fact that the same number of men and women join the meeting and did not take into consideration that there was originally more men than woman at the meeting.

4. **Link to RNCS**
   Learning outcome 1: The RNCS requires learners from grade 5 to 7 to be able to calculate by using operations appropriate to solve problems using addition and subtraction of common fractions (RNCS p 43 and p 70).

M012016

In which of these pairs of numbers is 2.25 larger than the first number but smaller than the second number?

Ⓐ 1 and 2

Ⓑ 2 and $\frac{5}{2}$

Ⓒ $\frac{5}{2}$ and $\frac{11}{4}$

Ⓓ $\frac{11}{4}$ and 3

Content Domain: **Number**

Main Topic: **Fractions and decimals**

Cognitive Domain: **Knowing Facts and Procedures**

Key: **B**

1. **Responses of learners:**

A: 17.7%, B: 34.3% C: 18.4% D: 10.2%
Attempt made but not correct (double response for MC items) 2.6%
Item omitted 7.5% Item not reached 9.2%

B is the correct answer

2. **How to solve the problem**

This item assesses learners' knowledge on pairs of numbers bracketing 2,25, and involves three steps.

Step 1: Convert $\frac{5}{2}$ to a decimal number i.e. 2.5.

Step 2: Convert $\frac{11}{4}$ to a decimal number i.e. 3.25

Step 3: Compare 2,25 with the two numbers in each possible answer to determine if it lies between the two numbers on the number line (true for option B).

3. **Errors of learners**

More or less equal number of learners selects the three incorrect answers. Learners that selected either C or D may have problems to convert common fractions to decimal fractions while all three wrong answers may indicate that learners did not know the place value of decimal numbers and are not able to order decimal numbers.

4. **Link to RNCS**

Learning outcome 1: The RNCS requires grade 6 learners to be able to recognises, use and represents decimal fractions to at lease two decimal places in order to describe and compare them and to use common fractions with 1-digit or 2-digit denominations (RNCS p 41).

M012027

What fraction of an hour has passed between 1:10 a.m. and 1:30 a.m.?

Ⓐ $\frac{1}{5}$

Ⓑ $\frac{1}{3}$

Ⓒ $\frac{1}{2}$

Ⓓ $\frac{2}{3}$

Ⓔ $\frac{3}{4}$

Content Domain: **Number**

Main Topic: **Fractions and decimals**

Cognitive Domain: **Solving Routine Problems**

Key: **B**

1. **Responses of learners:**

A: 11.0%, B: 21.7% C: 22.2% D: 11.8% E: 7.7%
Attempt made but not correct (double response for MC items) 3.5%
Item omitted 8.8% Item not reached 13.4%

B is the correct answer

2. **How to solve the problem**

This item, in which learners had to determine the fraction of an hour between two time points, assesses learners' knowledge about time and common fractions. There are four steps.

Step 1: Calculate the number of minutes that pass between 1:10 and 1:30 by subtracting 1:10 from 1:30 or by adding on from 1:10 to 1:30, i.e. 20 minutes.

Step 2: The number of minutes in 1 hour are 60.

Step 3: Express 20 minutes as a common fraction of 60 minutes, i.e. $\frac{20}{60}$.

Step 4: Simplify $\frac{20}{60} = \frac{1}{3}$

3. **Errors of learners**

The most frequent error made by learners was selecting option C as the correct answer with 22.2% of the learners selecting this option These learners either calculate the number of minutes that pass between 1:10 and 1:30 incorrect to get 30 minutes which will give a fraction of $\frac{30}{60} = \frac{1}{2}$, or they made a mistake when simplifying the fraction $\frac{20}{60}$.

4. Link to RNCS

Learning outcome 1: The RNCS requires learners to be able to solve problems involving calculations with minutes and hours in Grade 3 (RNCS p 29) and to be able to solve problems that use common fractions in Grade 7 (RNCS p 68).

M022199

$\frac{3}{5}+(\frac{3}{10}\times\frac{4}{15})=$

Ⓐ $\frac{3}{51}$

Ⓑ $\frac{1}{6}$

Ⓒ $\frac{6}{25}$

Ⓓ $\frac{11}{25}$

Ⓔ $\frac{17}{25}$

Content Domain: **Number**

Main Topic: **Fractions and decimals**

Cognitive Domain: **Knowing Facts and Procedures**

Key: **E**

1. **Responses of learners:**
   A: 14.9%, B: 7.0% C: 16.8% D: 15.1% E: 15.0%
   Attempt made but not correct (double response for MC items) 1.2%
   Item omitted 10.4% Item not reached 19.6%

   E is the correct answer

2. **How to solve the problem**
   This item assesses learners' knowledge about operations with fractions. The solution involves three steps.

   Step 1: Multiply $\frac{3}{10}$ with $\frac{4}{15}$ to get $\frac{2}{25}$.

   Step 2: To add $\frac{3}{5}$ and $\frac{2}{25}$ find the lowest common multiple of the denominators for $\frac{3}{5}$ and $\frac{2}{25}$ i.e. 25.

   Step 3: Express $\frac{3}{5}$ as a fraction with 25 as denominator, i.e. $\frac{15}{25}$.

   Step 5: Add $\frac{15}{25}$ and $\frac{2}{25}$ to get the $\frac{17}{25}$ represented by E.

3. **Errors of learners**
   The percentage of learners that selected option C (16.8%) and option D (15.1%) is both higher than the percentage of learners that selected the correct answer option B (15.0%). It appears that learners who selected option C first add the $\frac{3}{5}$ to $\frac{3}{10}$ and then multiply the answer by $\frac{4}{15}$. Learners that selected option D might have multiplied by $\frac{3}{5}$ instead of $\frac{5}{5}$ in step 3 when they express $\frac{3}{5}$ as a fraction with 25 as denominator to get $\frac{9}{25}$. They then add $\frac{9}{25}$ to $\frac{2}{25}$ to get the answer $\frac{11}{25}$.

4. **Link to RNCS**

Learning outcome 1, The RNCS requires Grade 7 learners to be to solve problems using addition, subtraction and multiplication of common fractions (RNCS p 70).

M022004

A teacher and a doctor each have 45 books. If $\frac{4}{5}$ of the teacher's books and $\frac{2}{3}$ of the doctor's books are novels, how many more novels does the teacher have than the doctor?

Ⓐ 2
Ⓑ 3
Ⓒ 6
Ⓓ 30
Ⓔ 36

Content Domain: **Number**

Main Topic: **Fractions and decimals**

Cognitive Domain: **Solving Routine Problems**

Key: C

1. **Responses of learners:**
   A: 14.2%, B: 8.2% C: 24.0% D: 21.5% E: 15.7%
   Attempt made but not correct (double response for MC items) 2.7%
   Item omitted 5.6% Item not reached 8.2%

   C is the correct answer

2. **How to solve the problem**
   This is a multi-step word problem making use of operations. There are three steps:
   Step 1: Multiply $\frac{4}{5}$ with 45 to determine the number of the teacher's novels, i.e. 36.
   Step 2: Multiply $\frac{2}{3}$ with 45 to determine the number of the doctor's novels, i.e. 30.
   Step 3: Subtract 30 from 36 to find the difference between their novels, i.e. 6.

3. **Errors of learners**
   The most frequent errors made by learners were selecting option A, D or E as the correct answer with respectively 14.2%, 21.5% and 15.7% of the learners selecting these options It appears that learners that selected option A may have simply subtract the numerators of the two fractions, therefore get 4 – 2 = 2. The learners that selected option D worked out only the number of the teacher's novels while the learners that selected option E worked out only the number of doctor's novels. In both options D and E the learners did not work out the difference between the two sets of novels.

4. **Link to RNCS**
   Learning outcome 1: The RNCS requires grade 7 learners to solve problems using addition, subtraction and multiplication of common fractions (RNCS p 70).

M022127

A car has a fuel tank that holds 45 L of fuel. The car consumes 8.5 L of fuel for each 100 km driven. A trip of 350 km was started with a full tank of fuel. How much remained in the tank at the end of the trip?

(A) 15.25 L
(B) 16.25 L
(C) 24.75 L
(D) 29.75 L

Content Domain
Number

Main Topic
Fractions and decimals

Cognitive Domain
Solving Routine Problems

Key
A

1. **Responses of learners:**
   A: 15.0%, B: 18.1% C: 22.4% D: 15.9%
   Attempt made but not correct (double response for MC items) 1.2%
   Item omitted 5.1% Item not reached 22.2%

   A is the correct answer

2. **How to solve the problem**
   This is a multi-step word problem making use of operations. The solution of the problem involves two steps:
   Step 1: Find the quantity of fuel that the car will consume on a trip of 350 km, e.g.
   $3 \times 8.5\,\ell + \frac{1}{2} \times 8.5\,\ell = 29.75\,\ell$
   Step 2: Find the difference between the quantity of fuel that the fuel tank holds and the quantity of fuel that the car will consume on a trip of 350 km, i.e. $45\,\ell - 29.75\,\ell = 15.25\,\ell$

3. **Errors of learners**
   The percentage of learners that selected the three incorrect options, option B 18.1%, option C (22.4%) and option D (15.97%) are higher than the percentage of learners that selected the correct answer, option A (15.0%). Learners that selected option B made a mistake in step 2. They borrow from the units to be able to subtract .86 from .30, but forget to reduce the units with the one unit that they borrowed. They then get an answer of $16.25\,\ell$. It appears that learners that selected option C worked out the petrol consumed for 250 km to get $21.25\,\ell$. When they worked out how much petrol remained in the tank they also forget to reduce the units with the one unit that they borrowed when they subtract the decimal part of the numbers. Learners that selected option D worked out the number of liters used but did not complete the problem to work out the liters that remained in the tank.

4. Link to RNCS

Learning outcome 1: The RNCS requires grade 7 learners to be able to calculate by using operations appropriate to solve problems using addition, subtraction and multiplication of positive decimals to at least 2 decimal places. (RNCS p.70).

M032079

John and Cathy were told to divide a number by 100. By mistake John multiplied the number by 100 and obtained an answer of 450.
Cathy correctly divided the number by 100. What was her answer?

(A) 0.0045
(B) 0.045
(C) 0.45
(D) 4.5

Content Domain
Number

Main Topic
Fractions and decimals

Cognitive Domain
Solving Routine Problems

Key
B

1. **Responses of learners:**
   A: 13.8%, B: 20.1% C: 28.8% D: 29.7%
   Attempt made but not correct (double response for MC items) 1.5%
   Item omitted 5.8% Item not reached 0.2%

   B is the correct answer

2. **How to solve the problem**
   The item requires knowledge of appropriate operations to solve problems with decimal numbers and involves two steps.
   Step 1: Find the original number that John use by applying the inverse operation of multiplication, i.e. $450 \div 100 = 4.5$
   Step 2: Find Cathhy's answer by dividing 4.5 by 100 to get 0.045.

3. **Errors of learners**
   The percentage of learners that selected option C (28.8%) and option D (29.7%) is both higher than the percentage of learners that selected the correct answer (option B with 20.1%). It appears that learners who selected options C and D did not grasp the concept of place value in decimal numbers and/or that they cannot multiply and divide correctly with decimal numbers. A smaller number of learners (13.8%) selected option A, were learners probable got the answer by dividing 450 twice by 100 which indicated that they were unable to select the correct operation to work out Cathys' number.

4. **Link to RNCS**
   Learning outcome 1: The RNCS requires grade 7 and 8 learners to be able to solve problems that involve multiple operations with rational numbers (including divisions with decimals) (RNCS p 71).

M012004

Alice can run 4 laps around a track in the same time that Carol can run 3 laps. When Carol has run 12 laps, how many laps has Alice run?

Ⓐ 9
Ⓑ 11
Ⓒ 13
Ⓓ 16

Content Domain
Number

Main Topic
Ratio, proportions, and percent

Cognitive Domain
Solving Routine Problems

Key
D

1. **Responses of learners:**

A: 13.4%, B: 9.0% C: 21.2 D: 42.4%
Attempt made but not correct (double response for MC items) 0.9%
Item omitted 5.1% Item not reached 8.1%

D is the correct answer

2. **How to solve the problem**

This item is based on concepts of ratio and proportion where learners had to find the number of laps that Alice ran in equivalent ratios. There are four steps:

Step 1: Write down the ratio of the laps run by Alice in relation to the laps ran by Carol, i.e. 4:3 or $\frac{4}{3}$.

Step 2: Write down the ratio of the laps run by Alice in relation to the laps ran by Carol if Alice ran 12 laps and the number of laps ran by Carol is represented by x, i.e. 12: x or $\frac{x}{12}$.

Step 3: The two ratios are equivalent therefore $\frac{4}{3}=\frac{x}{12}$.

Step 4: Solve for x by multiplying with 12 on both sides i.e. $\frac{4}{3}=\frac{x}{12} \rightarrow x = 16$

3. **Errors of learners**

The most frequent error made by learners was selecting option C as the correct answer with 21.2% of the learners selecting this option. The learners that selected this option may have reached the answer by finding the difference between values in the original ratio 4:3, i.e. 4 –3 = 1 and then add the 1 to the 12 laps that Carol ran to get the answer of 13.

4. **Link to RNCS**

Learning outcome 1: The RNCS requires grade 7 and 8 learners to be able to solve problems that represent ratio (RNCS p 70 and 71) and to solve problems using multiplication of common fractions (RNCS p 70).

M032727

Three brothers, Bob, Dan, and Mark, receive a gift of 45 000 zeds from their father. The money is shared between the brothers in proportion to the number of children each one has. Bob has 2 children, Dan has 3 children, and Mark has 4 children.

How many zeds does Mark get?

Ⓐ 5000
Ⓑ 10 000
Ⓒ 15 000
Ⓓ 20 000

Content Domain
Number

Main Topic
Ratio, proportions, and percent

Cognitive Domain
Solving Routine Problems

Key
D

1. **Responses of learners:**

A: 23.1%, B: 14.0% C: 24.4% D: 26.5%
Attempt made but not correct (double response for MC items) 0.8%
Item omitted 5.6% Item not reached 5.7%

D is the correct answer

2. **How to solve the problem**

This is a multi-step word problem making use of proportion and operations with common fractions. The solution to the problem involves three steps.

Step 1: Find the total number of children, i.e. 2 + 3 + 4 = 9.

Step 2: Express the number of Mark's children as a proportion of the total number of children, i.e. $\frac{4}{9}$

Step 3: Find the amount of zeds that Mark will receive by multiplying the proportion his children to the total number of children with 45 000 zeds, i.e. $\frac{4}{9} \times \frac{45\,000}{1} = 20\,000$ zeds

Therefore Mark will receive 20 000 zeds.

3. **Errors of learners**

The most frequent errors made by learners were selecting option A or C as the correct answer with respectively 23.1% and 24.4% of the learners selecting these options. It appears that learners that selected option A may worked out the total number of children correctly as 9 but then calculated the amount of zeds that will be received for one child, i.e. $\frac{1}{9} \times \frac{45\,000}{1} = 5\,000$ zeds. Learners that selected option C calculated the amount of zeds that will be received for three children (the number of Dan's children), i.e. $\frac{3}{9} \times \frac{45\,000}{1} = 15\,000$ zeds.

4. **Link to RNCS**

Learning outcome 1: The RNCS requires grade 9 learners to be able to solve problems that involve proportion and operations with common fractions (RNCS p 71)

M012017

Matchsticks are arranged as shown in the figures.

Figure 1 Figure 2 Figure 3

If the pattern is continued, how many matchsticks would be used to make Figure 10?

(A) 30
(B) 33
(C) 36
(D) 39
(E) 42

Content Domain
Algebra

Main Topic
Patterns

Cognitive Domain
Reasoning

Key
B

1. **Responses of learners:**
   A: 29.4%, B: 19.2% C: 11.6% D: 4.9% E: 17.4%
   Attempt made but not correct (double response for MC items) 1.4%
   Item omitted 5.7% Item not reached 10.3%

   B is the correct answer

2. **How to solve the problem**
   The item requires skills and ability to Investigates and extends geometric patterns to find the number of matchsticks used to make the tenth figure. There are three steps.
   Step 1: Find the number of matchsticks used in each figure shown, i.e. 6 for figure 1, 9 for figure 2 and 12 for figure 3.
   Step 2: Find the equation that will represent the pattern for the figures of matchsticks, i.e. 3n + 3
   Step 3: Find the number of matchsticks used in the tenth figure, i.e.
   3n+3 = 3(10) +3 = 33
   Therefore the number of matchsticks used in the tenth figure is 33.

3. **Errors of learners**
   The most frequent errors made by learners were selecting options A and E as the correct answer. Learners that had selected option A may have simply used the expression 3n leading to an answer of 30 matchsticks for figure 10. Learners who had multiplied 4 with 10 and add 2 (using the expression 4n + 2) would have reached the answer of 42 as the number of matchsticks for figure 10, therefore selecting option E as their answer.

4. **Link to RNCS**
   Learning outcome 2: The RNCS requires learners from grade 4 to grade 8 to be able to Investigates and extends geometric patterns (including patterns represented in physical or diagrammatic form (RNCS p 46, 47, 74 and 75).