

# TIMSS

## ITEM DIAGNOSTIC REPORT:

### SOUTH AFRICA

#### GRADE 9 MATHEMATICS



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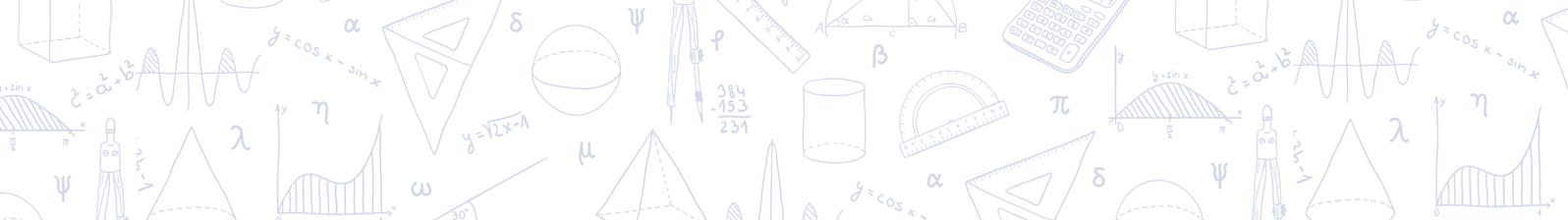
The Human Sciences Research Council (HSRC) released the results of the 2015 **Trends in International Mathematics and Science Study (TIMSS)** on 29 November 2016. TIMSS is a cross-national assessment of the mathematics and science levels of learners from the various participating countries. TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to allow participating nations to compare learner educational achievement across borders.



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# Preface

The Trends in International Mathematics and Science Study (TIMSS) is a four-year project. The 2015 TIMSS was carried out by the Education and Skills Development research programme of the Human Sciences Research Council under the direction of Dr Vijay Reddy. The study was funded by the Department of Basic Education.

This TIMSS Item Diagnostic report is one of three educator resource documents. The three reports contain diagnostic analyses of Grade 9 Mathematics TIMSS items; Grade 9 Science TIMSS items and Grade 5 Numeracy TIMSS items. Two reports containing the highlights of the Grade 5 and 9 TIMSS results have also been published. These documents, together with additional resources, are available on the TIMSS SA website: [www.timss\\_sa.org.za](http://www.timss_sa.org.za).

The authors would like to state their appreciation towards the Department of Basic Education for allowing access to learners in the school system in this way; all the individuals who assisted in the construction of this report through their valuable insight and comments during the course of two workshops.

We hope that this report will further develop the capacity of our educators to benefit our learners in achieving their potential, thereby contributing to the science and innovation system, world of work and societal well-being in the longer term.

This TIMSS Item Diagnostic report was compiled by Professor Mogege Mosimege, Ms Unathi Beku, Dr Andrea Juan, Ms Sylvia Hannan, Dr Casper H Prinsloo, Ms Jaqueline Harvey and Ms Ncamisile Zulu.

**Dr Vijay Reddy**

*Principal Investigator of TIMSS, South Africa*

## How can TIMSS 2015 help mathematics teachers and learners?

This document seeks to answer that question and is best described as a **educator resource document**. It draws conclusions from the TIMSS 2015 Grade 9 Mathematics assessment results to show common learner strengths and weaknesses and makes recommendations. It acts as a supplement and is designed to assist teachers and Department of Basic Education (DBE) officials. The teacher resource document is organised into separate sections:

### INTRODUCTORY

- How can TIMSS 2015 help mathematics teachers and learners?
- How do I use this document?
- Introduction and background

### SECTION 1

- A summary of common causal factors specific to South African education
- How to use the icons to link contextual factors and TIMSS 2015 items

### SECTION 2

- Overall weaknesses identified

### SECTION 3

- Overall strengths identified

### SECTION 4

- Item-by-item analysis

### SECTION 5

- Remedial actions

### SECTION 6

- Conclusion

### FINAL NOTES

- Appendix
- References

## How do I use this document?

This document can be used by all teachers, although it is specifically about Grade 9 learners. It does not replace or contradict any official DBE policies or documents, particularly those related to assessment and the delivery of the intended curriculum. When a teacher or DBE official receives this resource document, an easy way to navigate it is through the steps below. This sequence of steps has been outlined to assist teachers in helping their learners and/or assist DBE officials in their mentoring, coaching, training, and support of teachers.

### STEP 01

- Scan the table of contents to familiarise yourself with the document

### STEP 02

- Work through the summarised causal factors
- These factors were listed because they contribute to learner achievement on almost all items (the icons in the item-by-item analysis correspond to each factor)

### STEP 03

- Work through the overall strengths and weaknesses
- Think about how these affect your learners and your teaching

### STEP 04

- Work through the remedial recommendations
- Think about which recommendations are under your control, which ones you can refer to other agents, and which ones are outside of your control

### STEP 05

- Work through the item-by-item analysis
- Item information is sequenced by strand and topic, in that order
- Both multiple-choice and free-response (open-ended) items are reported

### STEP 06

- Based on the item-by-item analysis, identify and pursue remedial actions specific to your learners and your school
- This can be at the individual, item, or strand level

The contents page and rest of the document now follows. We hope that you find it informative and useful in teaching and assessment.

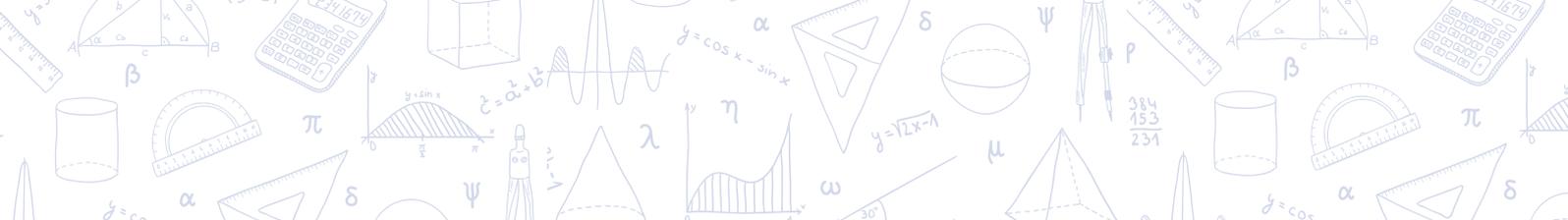


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# Introduction and background

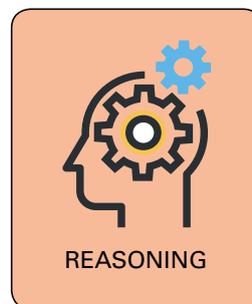
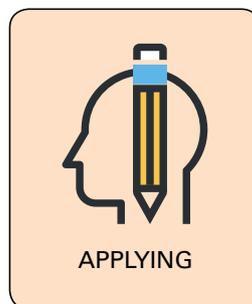
The Trends in International Mathematics and Science Study (TIMSS) assesses the mathematics and science knowledge of fourth grade and eighth grade learners around the world. TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to allow participating nations to compare learners' educational achievement across borders. TIMSS was first administered in 1995, and every four years thereafter. Fifty-nine countries and 425 000 learners around the world participated in TIMSS 2015.

The Human Sciences Research Council (HSRC) conducted TIMSS Mathematics and Science in South Africa in 292 schools among 12 500 Grade 9 learners. Participating in TIMSS assessments provides important information for a country's policy makers about the state of mathematics and science education and what is required for improving the quality of education. The assessment results therefore provide the data necessary for monitoring and evaluation, from which recommendations can be made to improve the health of a country's education system.

In order to facilitate comparison across countries and over time, TIMSS data are scaled to an international centre point (500) and standard deviation (100). This means that a score of 500 reflects the midpoint for all TIMSS country average scores. The TIMSS approach to scaling the achievement data is based on item response theory (IRT) scaling, which in turn is based on psychometric models, with marginal estimation. Through this methodology it is possible to include South Africa (TIMSS Mathematics and Science) on the same scale as the countries that participated in the full TIMSS assessment (TIMSS Grade 8).

The TIMSS 2015 diagnostic report for South African learner achievement in mathematics is organised into seven separate sections. **Section 1** deals with the common contextual factors that may explain patterns related to learner responses. These factors range from issues of language to the coverage of the content areas in mathematics. Although the identified factors can be considered central and critical in all aspects of education, and thus in learner achievement on all released items discussed in this report, they are only indicated in the item analysis when particularly salient, using their icons. This is in order to alert the reader without unnecessary repetition.

**Section 2** discusses the overall weaknesses identified in terms of the TIMSS content domains (listed above), TIMSS cognitive domains, and TIMSS item types (multiple-choice or open-ended). The TIMSS assessment framework utilises three cognitive domains: Knowing, Applying, and Reasoning. **Knowing** "refers to students' knowledge base of science facts, information, concepts, and tools" (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009). Items assessing this cognitive domain tested ability to recall/recognise, define, describe, illustrate with examples, and demonstrate knowledge of scientific instruments (Mullis et al., 2009). **Applying** items "involve the direct application of knowledge and understanding of science in straightforward situations" (Mullis et al., 2009). TIMSS 2011 included the following question formats to assess this cognitive domain: compare/contrast/classify; use models; relate; interpret information; find solutions; and explain. The last cognitive domain, **Reasoning** is an extension of both the Knowledge and Applying cognitive domains. It requires learners to solve problems using both familiar and unfamiliar contexts in complex tasks. Items in the Reasoning cognitive domain included asking learners to analyse, integrate/synthesise, hypothesise/predict, design, draw conclusions, generalise, evaluate, and justify (Mullis et al., 2009). South African learners appear to have struggled most with application and reasoning items.



**Section 3** identifies the overall strengths identified that have been found during the analysis. Although Grade 9 learner achievement in mathematics was below the international standard for the majority of items, there seemed to be a general indication that more South African learners are able to grasp more cognitively challenging content. This may be related to teachers placing an emphasis on: covering all CAPS content, creatively assisting all learners who have diverse backgrounds (e.g. language, mathematical knowledge base), and integrating content across multiple grades and subjects. However, further research into intrinsic and extrinsic factors impacting science teaching and learning is required.

**Section 4** provides the item-by-item analysis, a detailed discussion with exemplars and recommendations related to each question. Items are grouped into sub-sections according to the content area they are designed to assess, i.e., Number, Algebra, Geometry, and Data and chance. Within each sub-section, a statistical item analysis of each released item is presented in table format (percentage values are given for each coded item response). There are two types of items: multiple-choice and open-ended. For the multiple-choice questions, a comparison is made across item response selection for South African learners as well as with regard to the international standard (the correct answer for the multiple-choice questions is indicated in the tables). During analysis of the open-ended questions, a comparison is made according to the diagnostic code assigned when the item was marked. Each item, regardless of type, is then discussed in terms of learner performance, the South African curriculum, the TIMSS cognitive process, types of errors, and recommendations.

**Section 5** discusses remedial measures based on the strengths and weaknesses identified in this report with particular reference to the factors discussed in the first section. Previous research which has assessed the usefulness of alternative policy implementation as well as teaching approaches and practices are referenced here as well.

In **Section 6**, conclusions are provided. This section also discusses the importance of problem solving in mathematics which is regarded as critical in the understanding of how learners engage in various tasks and how this affects their performance. It suggests how problem-solving processes should be introduced and integrated into mathematics classroom activities in all content areas.

We hope that you find this document useful in your daily practices.

# 1. Common contextual factors and icons

Common South African contextual factors that may explain patterns related to learner responses are sequenced below chronologically. The sequence reflects:

- how certain whole-system structural/policy parameters precede learners' schooling, not least of which is overall language policy and its implementation **(Factor one)**;
- that learners arrive at Grade 9 with certain foundational deficits **(Factors two and three)**;
- that learners expect tuition in the intended policy-based curriculum **(Factor three)**;
- that teachers have diverse levels of training in their subject and its related instructional proficiencies and that teachers achieve variable coverage of the scheduled work **(Factors four and six)**; and
- that learners require training in certain testing techniques **(Factor five)**.

## Factor one: Language



South African learners are unable to read with understanding. They struggle with digesting comprehensive information and sifting through it for key information. Furthermore, they are unable to express their thoughts in words and structuring of ideas in a logical manner. They also struggle with sentence structures. Mathematics is in itself a language. South African learners have difficulty understanding the mathematical language and mathematical terms such as “and”, “of”, “greater than” and “less than”, amongst others.

## Factor two: Back to basics



South African learners struggle with displaying knowledge and understanding of basic mathematical concepts. This could point to teaching challenges within the foundation and intermediate phases of the schooling system. Senior phase teachers often have to reteach basic concepts before progressing to abstract ideas.

## Factor three: Curriculum



One of the difficulties is the lack of gradual progression and thread with the curriculum. The CAPS curriculum is too packed. Senior phase teachers struggle to get through the content because they often have to reteach basic concepts. This impacts the depth of teaching and the amount of time dedicated to exercises.



## Factor four: Teacher training and support



There is a need for constant teacher training on practical classroom exercises that can solidify understanding of basic concepts. Furthermore, there is recognition of the need for extra textbook resources to complement the standard CAPS curriculum textbook. The reintroduction of geometry into the curriculum requires that teachers be retrained.

## Factor five: Exam and testing techniques



South African learners experienced difficulty in answering multiple-choice questions. They failed to apply the appropriate steps such as calculating before selecting an option, and eliminating incorrect options before deciding on the correct one. Multiple-choice questions are often posed as phrases that contain qualifiers. In order to comprehend the intended meaning of these phrases, it is expected or assumed that learners have a good understanding of the English and mathematical languages. As noted in Factor one, South African learners have difficulty reading, writing and understanding the intended meanings of the English and mathematical languages. Therefore, they are further disadvantaged when it comes to testing techniques (such as multiple-choice questions) that require the application of language skills.

## Factor six: Coverage of TIMSS topics by assessed South African classes



The subject of mathematics comprises a series of topics that progress from easiest, basic concepts to difficult, complex concepts. It is necessary that curriculum and classroom teaching also follow this progression so that an adequate foundation is laid and learners can build on the acquired knowledge. The following table (Table 1) displays each TIMSS Grade 9 Mathematics topic and the degree (percentages) at which it was covered before, during and after Grade 9:

## Common contextual factors and icons (continued)

**Table 1: Coverage of TIMSS 2015 Mathematics curriculum topics as reported by teachers**

TIMSS Mathematics topics	Mostly taught before this year	Mostly taught this year	Not yet taught or just introduced
<b>Number</b>			
Computing with whole numbers	59,8	35,0	5,3
Comparing and ordering rational numbers	49,4	49,9	0,6
Computing with rational numbers (fractions, decimals, and integers)	51,6	45,7	2,8
Concepts of irrational numbers	41,5	56,3	2,2
Problem solving involving percents or proportions	46,9	51,1	2,0
<b>Algebra</b>			
Simplifying and evaluating algebraic expressions	34,8	63,6	1,6
Simple linear equations and inequalities	24,5	60,1	15,5
Simultaneous (two variables) equations	11,9	37,4	50,7
Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalisation of patterns)	34,6	61,0	4,4
Representation of functions as ordered pairs, tables, graphs, words, or equations	25,1	61,8	13,1
Properties of functions (slopes, intercepts, etc.)	13,7	49,4	36,9
<b>Geometry</b>			
Geometric properties of angles and geometric shapes (triangles, quadrilaterals, and other common polygons)	41,1	56,8	2,2
Congruent figures and similar triangles	33,5	63,2	3,3
Relationship between three-dimensional shapes and their two-dimensional representations	28,2	47,1	24,7
Using appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes	3,5	52,4	13,1
Points on the Cartesian plane	26,0	52,8	21,2
Translation, reflection, and rotation	20,9	27,0	52,1

## 2. Overall weaknesses identified

### TIMSS content domains

The South African learners performed lower than the average of their international counterparts in all four content domains, as depicted in Table 2.

**Table 2: Content area achievement comparison between South African and international learners (% correct) (all TIMSS items)**

	Number	Algebra	Geometry	Data and chance
South Africa	21	20	19	26
International	44	37	37	47

The performance in Data and chance is higher than in all the other content domains, including in the area of Number. It is critical to note that in the South African curriculum, Data handling (the equivalent of the TIMSS content area of Data and chance) is mostly introduced in Grades 7 to 9, whereas a content area like Number, operations and relationships (the equivalent of the TIMSS content area of Number) is introduced at grades that are lower than the senior phase. It could be reasonably expected that since some of the work on Numbers is introduced in lower grades (like Grade 6) and is therefore revised in Grades 8 and 9, the learners would perform even higher in this content domain. However, the results show that this is not the case. The comparison table (Table 2) suggests that learners seem to have a better understanding of Data handling, followed by Number, patterns and relationships. Performance in the content area of Patterns, functions and algebra (the equivalent of TIMSS content area of Algebra) is the lowest at 20,2%. However, this is also the case in comparison with the average performance at the international level.

The comparison in Table 2 regarding Algebra and Geometry states that these two content areas are difficult for all learners (lowest percentages).

### TIMSS cognitive domains

Table 3 shows that the learners performed better in the cognitive domain of Knowing, followed by the cognitive domain of Applying and then the cognitive domain of Reasoning. This is not unusual as the cognitive domain of Knowing requires the learners to deal with the content at a fairly basic and uncomplicated level as compared to the cognitive domain of Reasoning. It is also interesting to note that the learners performed on average the highest in the cognitive domain of Applying rather than in the cognitive domain of Knowing, although the performance is consistent with the performance noticed in the Data and chance content domain.

**Table 3: Cognitive domain achievement comparison between South African and international learners (% correct) (all TIMSS items)**

	Knowing	Applying	Reasoning
South Africa	30	19	15
International	50	40	32

## Overall weaknesses identified (continued)

### TIMSS type of questions

The performance of the learners in the multiple-choice questions is comparable to the performance in the constructed-response items, as can be seen from Table 4. This is a little unusual as it is expected that learners would find it hard to construct their own responses as compared to selecting from multiple choices. Perhaps the conclusion that can be drawn is that the questions asked in both multiple-choice and constructed-response questions were able to reveal, to a similar extent, the difficulties that the learners experienced in specific content areas. The only differences would be in the content domain of Number, in which the learners performed higher in constructed-response items than in the multiple-choice items.

It must be noted that a vast majority of learners have no idea how to answer multiple-choice questions as it is not a common way of asking questions in South African Senior (FET) mathematics papers.

**Table 4: Item type achievement comparison between SA and international learners (% correct) (Limited use items only)**

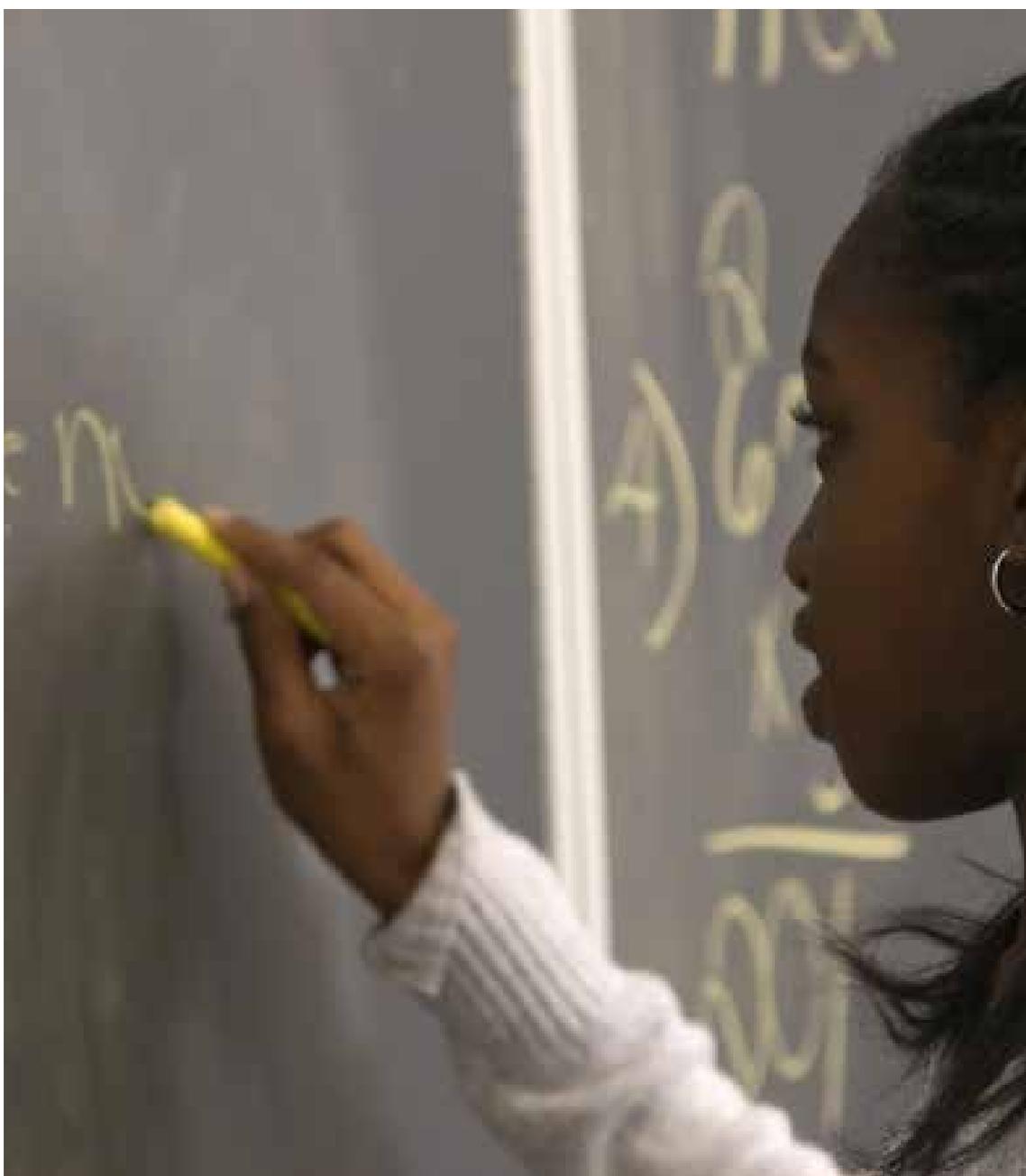
	Multiple-choice questions	
	South Africa	International average
Number	16,85	42,2
Algebra	22,6	39,1
Geometry	20,0	41,9
Data and chance	37,5	55,1

	Constructed-response questions	
	South Africa	International average
Number	33,3	54,1
Algebra	18,3	35,8
Geometry	24,8	44,5
Data and chance	12,5	36,4



### 3. Overall strengths identified

It is noted that the performance of the learners in content areas that were covered in the period before the TIMSS data collection in August 2015, especially the content areas covered close to the data collection (the period between July and August 2015), were performed far better than the content areas that were covered after the collection of the data. This can be seen where mentioned specifically in the item-by-item analysis section. This may require that South Africa consider the period in which the data is collected in relation to the coverage of the specified content.



Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

# 4. Item-by-item analysis

Each released (limited-use) item is reviewed first by cognitive domain and then by content domain. The items are discussed in terms of learner performance, learner answers, South African curriculum (CAPS) location, TIMSS cognitive domains, error analysis, and recommendations. It is noted that icons representing overarching contextual factors are inserted alongside each item number (code) and label. The discussion of these factors in Section 1 also included recommendations which are accordingly not repeated here. Further recommendations made in this section are therefore specific to each item.

The questions are arranged and grouped according to the content area they are designed to assess based on the TIMSS 2015 Assessment Framework. The four Mathematics content areas are:

- Number
- Algebra
- Geometry, and
- Data and chance.

These four areas have been compared, as far as is possible, to the Mathematics Curriculum and Assessment Policy Statement for Senior Phase (Grades 7 – 9). It is necessary to indicate that in the identification of some of the content areas it was noted that there was no direct match between the TIMSS content areas and those of the senior phase in South Africa. In such cases, the closest content areas have been identified according to the analysis of the content of the question. In some instances, the content area in the senior phase has been found to cut across more than one area, and where such exists it has been identified accordingly.

Following these content domains, the questions have been further arranged according to relevant cognitive domain. Therefore, the questions have been arranged according to the content area and within each content area further arranged according to the cognitive domains of **Applying, Knowing and Reasoning**.

## TIMSS content area: Number

Cognitive area: Knowing

**QUESTION 1**



Write this as a decimal number.

$$8 + 50 + \frac{3}{100} + \frac{1}{10}$$



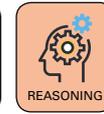
M01\_02

### Learner performance

	Correct	Incorrect	Omitted
South Africa	26,1	65,8	6,3
International average (n=38)	35,0	48,7	13,2

### South African curriculum

Whole numbers – Calculations with fractions (common fractions and decimal fractions).



### Types of errors

The correct answer for this question was 58,13. Two-thirds of Grade 9 learners gave an incorrect response. This indicates a lack of understanding of problems which involve working with fractions, especially when they are combined with whole numbers. Learners may have had trouble converting the fractions to decimal fractions.

### Recommended action

Teachers should focus on problems which involve fractions, ensuring that learners are able to solve problems which include both fractions and whole numbers. In addition, a focus on conversion between common fractions and decimal fractions is necessary.

Cognitive area: Knowing

QUESTION 2

Which of the following ratios is equivalent to the ratio 1:4?

- A. 4:16
- B. 4:7
- C. 4:5
- D. 4:1

### Learner performance

	A	B	C	D	Omitted
South Africa	27,4	3,6	5,7	62,7	0,5
International average (n=38)	59,9	4,6	4,7	29,0	1,7

### South African curriculum

Whole numbers – Ratio and rate.

### Types of errors

The majority of learners selected option D, suggesting that the concept of ratio has not been well understood. These learners have chosen the option which contains the same two numbers as in the original ratio (in reverse order). These learners may have misunderstood the term “equivalent;” and therefore chose an option which appeared to be the same as the original ratio. This incorrect response denotes a serious misconception in the area of ratios and proportion equivalence.

### Recommended action

Teachers should use practical examples for illustration to build conceptual understanding of ratios.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 3** 

For every whole number  $n$ , are these statements **true** or **false**?  

Fill in one circle for each statement.

	True	False	
$n + 4 = 4 + n$ _____	A	B	
$n - 5 = 5 - n$ _____	A	B	
$n \times 6 = 6 \times n$ _____	A	B	
$n \div 7 = 7 \div n$ _____	A	B	

M02\_01

### Learner performance

First line	Correct	Incorrect	Omitted
South Africa	81,2	15,7	2,8
International average (n=38)	85,2	10,2	4,2
Second line	Correct	Incorrect	Omitted
South Africa	46,7	47,8	5,2
International average (n=38)	70,4	23,9	5,3
Third line	Correct	Incorrect	Omitted
South Africa	67,3	26,5	5,9
International average (n=38)	76,8	17,6	5,3
Fourth line	Correct	Incorrect	Omitted
South Africa	46,8	46,8	6,1
International average (n=38)	70,2	23,8	5,7

### South African curriculum

Numbers, operations and relationships – Integers (commutative properties).

### Types of errors

For the first line, a high percentage of learners (81,2%) gave the correct response, which suggests that they understand commutative property of addition as they apply to variables. This suggests that this is an area of content that the learners are comfortable with. About three-quarters of learners answered the third line in the question, which incorporates multiplication, correctly. For the second and fourth lines, which incorporate subtraction and division, less than half of the learners answered correctly. This may indicate that although learners largely understand commutative properties as they relate to addition and multiplication, they become confused when this property is applied to subtraction and division. Learners may also have assumed that the two sides are equal as they contain the same variables. For addition and multiplication this would be the case, as the order of the variables does not change the answer. However, for subtraction and division, the order of the variables affects the answers.



### Recommended action

Commutative property still needs to be addressed as it creates confusion, especially as it is applied to some mathematical operations. Teachers therefore need to continue emphasising the commutative, associative and distributive properties, with an emphasis on subtraction and division. Classroom exercises and practical illustrations using numbers and then variables should be gradually introduced. For example;  $n + 5 \neq 5 + n$  should be preceded by a numbers examples such as  $5 - 1 \neq 1 - 5$ .

#### Cognitive area: Knowing

#### QUESTION 4



M03\_01

Which number is closest in size to  $\frac{3}{4}$ ?



- A. 0,34
- B. 0,43
- C. 0,74
- D. 0,79

### Learner performance

	A	B	C	D	Omitted
South Africa	48,8	10,1	37,3	3,1	0,6
International average (n=38)	22,8	9,5	61,5	4,8	1,2

### South African curriculum

Whole numbers – Calculations with common fractions and decimal fractions (conversion).

### Types of errors

The majority of learners (48,8%) chose option A rather than the correct option C, highlighting an example of confusion between common fractions and decimal fractions. It is interesting to note that the learners chose an option which has the same numbers as the common fraction, rather than the one with the closest value. This may illustrate a clear misconception in converting from common fractions to decimal fractions, as well as equivalent fractions.

### Recommended action

Teachers need to emphasise multiple-choice question techniques. While fractions are taught in the intermediate phase, there is a need for senior phase teachers to reteach fractions, or at least the related basic principles. Intermediate phase teachers need to strengthen conceptual teaching and understanding of equivalent fractions.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 5**  

Place <, >, or = in each box so that each statement is true.

 0,35  0,350  
0,35  0,4  
0,35  0,305  
0,35  0,035

M03\_014

### Learner performance

	Correct	Incorrect	Omitted
South Africa	8,3	91,1	0,6
International average (n=38)	41,7	57,1	0,9

### South African curriculum

Numbers, operations and relationships: Decimal fractions.

### Types of errors

Over 90% of South African learners answered this question incorrectly. This suggests that learners have difficulty with decimal fractions and their comparison. Learners may also have difficulty with the mathematical symbols.

### Recommended action

Teachers need to go back to the basics with respect to mathematical symbols and decimals, particularly comparing and ordering of decimal fractions. This knowledge needs to be consolidated in earlier grades, and teachers need to ensure that learners have the fundamental knowledge which can then be built on in more complex problems.

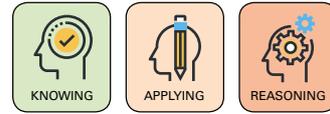
**Cognitive area: Knowing**

**QUESTION 6**   

What number must 202,6 be divided by to get an answer of 2,026?

 A. 0,01  
B. 0,1  
C. 10  
D. 100

M05\_01



### Learner performance

	A	B	C	D	Omitted
South Africa	18,5	22,4	12,6	44,7	1,6
International average (n=38)	14,8	14,0	9,0	60,6	1,3

### South African curriculum

Whole numbers – Decimal fractions (calculations with decimal fractions).

### Types of errors

Nearly half of the South African Grade 9 learners answered this question correctly. This may suggest a good understanding of division of decimals by hundreds. Those learners who answered the question incorrectly may have difficulty with decimals and associated calculations.

### Recommended action

Teachers need to emphasise the basic concepts of the decimal system and to provide learners with examples of calculations using decimals.

*Cognitive area: Knowing*

**QUESTION 7**



A shop advertises “20% off” the price of a book.

What fraction is the price reduced by?

A.  $\frac{1}{20}$       C.  $\frac{1}{4}$

B.  $\frac{1}{5}$         D.  $\frac{1}{2}$





M05\_02

### Learner performance

	A	B	C	D	Omitted
South Africa	49,1	31,4	10,1	7,6	1,4
International average (n=38)	25,7	57,5	9,7	5,2	1,7

### South African curriculum

Whole numbers – Calculations with common fractions and decimal fractions (conversion).

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Types of errors

The similarity between the way learners answered this question and the way they answered Question 4 is striking. In Question 4, the majority of learners (48,8%) chose the incorrect option which contained the same numbers as in the fraction provided. Similarly, when answering this question, nearly half of the learners chose the incorrect option A, in which there is a number “20” in the question (20%) and 20 in the answer. This strengthens the earlier conclusion about misconception in converting from common fractions to decimal fractions.

### Recommended action

Teachers need to reinforce learners’ understanding of equivalence, and provide examples using percentages and decimals.

*Cognitive area: Knowing*

**QUESTION 8** 

 **Approximate  $103 + 289 + 475 + 310 + 519$  to the nearest hundred.**

M05\_03

### Learner performance

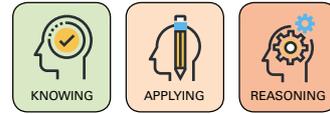
	Correct	Incorrect	Omitted
South Africa	17,0	73,2	6,0
International average (n=38)	40,3	47,2	8,4

### South African curriculum

Whole numbers – Ordering and comparing whole numbers (Grade 7).

### Types of errors

The correct answer for this question was 1 700. Nearly three-quarters of learners gave an incorrect answer, suggesting that the majority of learners are either struggling with the rounding-off concept or, more specifically, how to round off to the nearest hundred. However, it is also interesting to note that the performance at the international level is not very high, at 40,3%. The low performance by the learners therefore suggests that they have either forgotten what was taught in a lower grade, or they did not necessarily have a strong foundation in terms of this mathematical concept of rounding off.



### Recommended action

The concept of rounding off needs to be consolidated in lower grades in order to ensure that learners have a strong foundation when encountering more complex problems related to this concept. Recommended action can be linked to Factor three of the contextual factors, which highlights the importance of the thread in or successive progression of the curriculum.

Cognitive area: Knowing

**QUESTION 9**

Which of these expressions is equal to  $25 \times (16 - 11)$ ?

- A.  $(25 \times 16) - 11$
- B.  $(25 - 16) \times 11$
- C.  $(25 \times 11) - (25 \times 16)$
- D.  $(25 \times 16) - (25 \times 11)$

M06\_01

### Learner performance

	A	B	C	D	Omitted
South Africa	26,7	12,8	13,8	<b>44,8</b>	1,9
International average (n=38)	17,6	10,3	12,2	<b>57,7</b>	2,1

### South African curriculum

Whole numbers – Properties of whole numbers. This is actually covered in the Grade 6 Mathematics curriculum in which the learners are expected to “recognise and use the Commutative, Associative and Distributive Properties of Whole Numbers”.

### Types of errors

The majority of learners (close to half) chose the correct answer. However, 26,7% of the learners chose option A, which states:  $(25 \times 16) - 11$ . The choice of this option by so many learners suggests that they do not understand the distributive property in relation to mathematical symbols. So the choice could be an illustration of the difficulty in the two mathematical concepts of distributive property and mathematical symbols. The remediation would need to look at both mathematical concepts.

### Recommended action

Laying the proper foundation of basic mathematical principles such as distributive laws, commutative laws, and the application of BODMAS are crucial areas for teachers to focus on.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 10** Complete each fraction so that all the fractions are equivalent.   



$$\frac{24}{60} = \frac{6}{15} = \frac{2}{5} = \frac{\quad}{120} = \frac{16}{\quad}$$

M06\_04

### Learner performance

	Correct	Incorrect	Omitted
South Africa	7,0	85,0	7,9
International average (n=38)	35,9	49,6	14,1

### South African curriculum

Numbers, operations and relationships: Common fraction (Section 1.4). This is a section that is taught in Grade 6, so in Grade 9 it is stated as “Revise and Use the knowledge of Equivalent Fraction to add and subtract common fractions”.

### Types of errors

The correct answer for this question was  $\frac{(24)}{(60)} = \frac{(6)}{(15)} = \frac{(2)}{5} = \frac{48}{(120)} = \frac{(16)}{40}$ . Learners were required to complete all fractions correctly. Only 7% of the learners gave the correct response for all of the fractions, indicating a high level of misconception of equivalent fractions. Learners may have been able to complete some of the fractions, but encountered difficulty in completing the full set.

### Recommended action

A solid foundation is required for learners to be able to work with equivalent fractions. Examples should be provided of working with sets of equivalent fractions and ensuring that learners understand the underlying concepts. Revising teaching methods of fractions using wall charts from Grade 4 to Grade 6 may be useful.

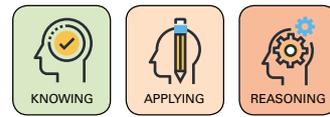
**Cognitive area: Knowing**

**QUESTION 11** What is the value of  $3^3$ ?   



A. 6  
B. 9  
C. 27  
D. 33

M07\_01



### Learner performance

	A	B	C	D	Omitted
South Africa	8,1	28,3	61,9	1,1	0,5
International average (n=38)	4,3	23,4	70,0	1,4	0,7

### South African curriculum

Numbers, operations and relationships: Exponents (Section 1.2). This is a section that is taught in Grade 7 and it deals with comparing and representing numbers in exponential form. In Grade 9 the relevant section deals with revision of comparison as well as representation of integers in exponential form.

### Types of errors

The performance of the learners in this question is quite high (61,9%), comparing favourably and closely with the international average (70,0%). The fact that 28,3% of the learners chose option B, which is 32, instead of the correct option C, which is 33, suggests that these learners still have trouble with representing numbers in exponential form. Learners who lack understanding of exponents may have multiplied 3x3 which would have given them an answer of 9 (option B). It should be noted that option B is a very plausible option and a good distractor in relation to the correct option C.

### Recommended action

Teachers need to focus on ensuring that learners understand how to do calculations involving exponents.

Cognitive area: Knowing

QUESTION 12

A team won 60% and tied 15% of the games it played.

What percent of games did the team lose?

M07\_03

### Learner performance

	Correct	Incorrect	Omitted
South Africa	25,5	72,8	1,6
International average (n=38)	64,6	31,1	4,1

### South African curriculum

Numbers, operations and relationships: Common fractions (Section 1.4). This is a section that is taught in Grade 7 and it deals with percentages – solving problems in contexts involving percentages; so in Grade 9 it is undertaken as revision of the work on percentages done in Grade 7 (actually started in Grade 6).

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Types of errors

The correct answer for this question was 25%. Nearly three-quarters of Grade 9 South African learners answered this question incorrectly. Although the question appears straightforward, learners may have misunderstood what was required in this word problem or may have been unsure of the correct calculations to use to answer the question. Figure 1 provides an example of an incorrect answer where a learner used fractions to try to answer the question.

**Figure 1:** Question 12 learner incorrect response example

A team won 60% and tied 15% of the games it played.  
What percent of games did the team lose?

Answer: 9 %

60% of 15%  
 $\frac{60}{100} \times \frac{15}{100}$   
 $= \frac{900}{10000}$   
 $= \frac{9}{100}$

“ 60% of 15%  
 $60/100 \times 15/100$   
 $= 900/10\ 000$   
 $= 9/100$ ”

### Recommended action

Classroom teaching needs to emphasise the solution of word problems that are related to real-life situations. Emphasis also needs to be placed on working with percentages.



### Cognitive area: Knowing

#### QUESTION 13 A



The table shows the number of pieces of paper in a stack and the height of the stack.



Number of pieces of paper in the stack	100	150	200
Height of the stack (mm)	8		

A. Complete the table.

M07\_04

### Cognitive area: Applying

#### QUESTION 13 B



B. How many pieces of paper would there be in a 28 mm high stack?

Answer: \_\_\_\_\_

M07\_04

### Learner performance

Question 13A	Correct	Incorrect	Omitted
South Africa	22,0	66,2	12,7
International average (n=38)	52,7	32,5	14,6

Question 13B	Correct	Incorrect	Omitted
South Africa	15,1	75,0	9,8
International average (n=38)	43,9	40,5	15,3

### South African curriculum

Numbers, operations and relationships.

Ratio and direct proportion: Equivalent fractions. Patterns.

Patterns, functions and algebra, especially functions and relations using tables.

### Types of errors

The correct responses for part A of the question were 12 and 16. The majority of learners (63,6%) did not get either of the two values correct. This may indicate learners' difficulty with solving word problems, or completing a table where one value is given and the learner is required to work out the remaining values based on the information provided.

The correct answer for part B was 350. Part B is a typical equations problem in which the solution can be established from a determined relationship. A high percentage of learners giving an incorrect answer suggests that the learners are struggling with equations, especially when they have to be determined from word problems.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Recommended action

It is important for teaching to emphasise the solution of word problems, particularly those that are related to real-life situations. Furthermore, teachers should equip learners with the skills to be able to complete patterns based on the information that is given to them.

#### Cognitive area: Applying

#### QUESTION 14



In the long jump at the school sports day, Bongani jumped 4,8 metres.



John jumped  $\frac{3}{4}$  of this distance. How far did John jump?

- A. 1,2 m
- B. 1,6 m
- C. 3,6 m
- D. 6,4 m

M01\_01

### Learner performance

	A	B	C	D	Omitted
South Africa	22,3	14,0	39,2	21,5	3,0
International average (n=38)	11,1	10,9	64,1	11,2	2,5

### South African curriculum

Whole numbers – Calculations with fractions (common fractions and decimal fractions); Grade 8.

### Types of errors

Nearly 40% of South African learners chose the correct response, although this is lower than the international average of 64,1%. The fact that a significant number of learners (22,3%) chose the incorrect option A suggests that they are either struggling with fractions or did not know how to relate fractions to calculations of distance.

### Recommended action

Intermediate phase teachers should lay a good foundation of fractions and decimals. More exercises on the progression from a whole number to a decimal number should be taught in the classroom. Teachers should emphasise mathematics terminology to the learners. Furthermore, they should create the understanding that a quarter means dividing into four equal parts.

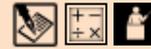


*Cognitive area: Applying*

**QUESTION 15 A/B**



Vuyo was going to buy a new Supertext cell phone.  
She looked at these two advertisements.



**Company X**

The New Supertext Cell Phone  
Get this great phone free!  
250 zeds monthly charge  
Calls 3 zeds per minute  
Text messages 2 zeds each

**Company Y**

The New Supertext Cell Phone  
Cheap rates for calls and texts!  
Buy the phone for 2 500 zeds  
Only 50 zeds monthly charge  
Calls only 2 zeds per minute  
Text messages only 1 zed each

Vuyo decided to compare how much it would cost to have the phone for a year without making any calls or sending any text messages.

A. Work out the cost of having the Supertext phone for a year from Company X and from Company Y.

Costs: Company X \_\_\_\_\_ Company Y \_\_\_\_\_

B. Vuyo then estimated how much she was likely to use the phone. She thought she would talk on the phone for 500 minutes in the first year and send 200 text messages. Find out how much she would pay for the phone in the first year from each company. Do not forget the monthly charge and other costs.

Costs: Company X \_\_\_\_\_ Company Y \_\_\_\_\_

M01\_06

*Cognitive area: Reasoning*

**QUESTION 15 C**



C. Which plan is less expensive for Vuyo?

Less expensive plan \_\_\_\_\_

Explain your answer in terms of the cost of the phone, the monthly charge, and the rates for the calls and text messaging.

M01\_06

**Learner performance**

Question 15A	Correct	Part A correct	Part B correct	Incorrect	Omitted
South Africa	4,8	16,0	0,1	73,5	5,6
International average (n=38)	31,5	15,1	1,2	40,9	11,0

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

Question 15B	Correct	Partially correct	Incorrect	Omitted
South Africa	4,2	4,6	79,4	11,3
International average (n=38)	31,8	12,9	42,8	16,3

Question 15C	Correct	Partially correct	Incorrect	Omitted
South Africa	3,0	4,7	81,7	10,0
International average (n=38)	9,1	9,1	66,8	14,6

### South African curriculum

Numbers, operations and relationships.

### Types of errors

#### Part A

The correct responses were 3 000 and 3 100. Partially correct responses were only 3 000 correct or only 3 100 correct. All other responses were incorrect. Some learners simply added the available numbers to calculate 255 and 2 553, respectively (even here some learners could not do the simple addition and wrote “25 521” rather than adding the 2 and 1 together). Other learners wrote down the cost per month for each phone without any further calculations. One learner (excerpt in Figure 2 below) multiplied both the one-off cost for the phone and the monthly charges by 12. This may indicate misunderstanding or a lack of careful reading of the question.

**Figure 2:** Question 15 Part A learner incorrect response example

A. Work out the cost of having the Supertext phone for a year from Company X and from Company Y.

Costs: Company X \_\_\_\_\_ Company Y \_\_\_\_\_

$$250 + 2$$

$$= 252 \times 12$$

$$= 3024$$

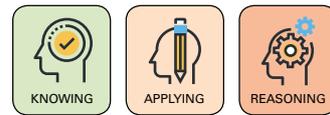
$$2500 + 50$$

$$= 3550 \times 12$$

$$= 30600$$

$250 + 2$	$2500 + 50$
$= 252 \times 12$	$= 2550 \times 12$
$= 3024$	$= 30600$

Several learners were able to grasp the principle, which was to multiply the cost per month by 12 to arrive at the total cost for the year. However, they did not remember to exclude cost for calls and/or messages as stated in the question. This indicates either inattention to detail, misunderstanding of the question, or a possible influence of language. The number of mistakes made on this item is concerning, as making comparisons between two purchases is a common real-life task.



A very high percentage of learners gave an incorrect answer. This may be attributed to an interpretation of a word problem, i.e. converting a word problem into a formula as well as giving a correct interpretation. This is a type of a problem that would also be applicable to real-life situations, which suggests that the learners cannot relate to or satisfactorily solve problems related to real-life situations.

**Part B**

Correct responses for part B were 4 900 and 4 300, or correct values based on the incorrect values determined by learners in part A of this question. Partially correct responses were only 4 900 correct, only 4 300 correct, or only one value correct but based on learners' incorrect values in part A. For learners whose responses were scored as partially correct, one learner forgot to take into account the total one-off cost of the phone and then added the calculated monthly charges and other costs to this amount (excerpt below). This indicates a lack of attention to detail and perhaps a focus on the explicit information rather than the implied information.

**Figure 3:** Question 15 Part B learner incorrect response example

$250 \times 12 = 3000 \text{ zeds}$ $= 3 \times 500 \text{ minutes}$ $= 1\,500 \text{ zeds}$ $2 \times 200 = 400 \text{ zeds}$ $= 3\,000 \text{ zeds} + 1\,500$ $\text{zeds} + 400 \text{ zeds}$ $= 4\,900 \text{ zeds}$	<p>Costs: Company X <u>4900 Zeds</u> Company Y <u>1800 zeds</u></p> $250 \times 12 = 3000 \text{ Zeds}$ $= 3 \times 500 \text{ minutes} = 1500 \text{ Zeds}$ $= 2 \times 200 = 400 \text{ Zeds}$ $= 3000 \text{ Zeds} + 1500 \text{ Zeds} + 400 \text{ zeds}$ $= 4900 \text{ Zeds}$ $50 \times 12 = 600 \text{ year charge}$ $= 2 \times 500 \text{ minutes}$ $= 1000 \text{ zeds per year}$ $= 1 \times 200 = 200 \text{ zeds - text messages a year}$ $= 600 + 1000 + 200$ $= 1800 \text{ Zeds}$	$50 \times 12$ $= 600 \text{ year charge}$ $= 2 \times 500 \text{ minutes}$ $= 1\,000 \text{ zeds per year}$ $= 1 \times 200$ $= 200 \text{ zeds text messages per year}$ $= 600 + 1\,000 + 200$ $= 1\,800 \text{ zeds}$
--	---	--

All other responses were incorrect. Some learners focused on which phone should be chosen rather than the calculations. Given the high percentage of incorrect responses in the first part of this question (Question M01-06A), it is not surprising that the percentage of incorrect responses in the follow-up question is even higher.

**Part C**

Correct response: Company Y [with explanation that points out all three of: (i) the phone is expensive/not free; (ii) the monthly charge is lower; (iii) the charges for calls and/or texts are lower. Partially correct with explicit reference to only two of: (i) the phone is expensive/not free; (ii) the monthly charge is lower; (iii) the charges for calls and/or texts are lower; or with explicit references to the phone being free and to both the higher monthly charge and higher rates for calls and/or texts, if the student's answers in part B give Company X as cheaper]

As in earlier questions a very high percentage of learners gave an incorrect answer. This is again suspected to be attributed to an interpretation of a word problem, i.e. converting a word problem into a formula as well as giving a correct interpretation. This problem is the type of problem that would also be applicable to real-life situations, which suggests that the learners have difficulty relating to or solving problems related to real-life situations.

## Item-by-item analysis (continued)

### Recommended action

Teachers need to teach or introduce word problems in primary school (foundation phase). Learners should be taught how to identify relevant information using key words. Learners should also be taught how to unfold and logically work through word problems. Teachers need to target the transfer of mathematical skills and application of concepts beyond the classroom to real-life applications. Classroom exercises should include complex word problems that are related to real-life situations. Questions of this nature should be included in classroom exercises and tests.

#### Cognitive area: Applying

#### QUESTION 16



Peter, Menzi and Sara have 150 zeds to share.

Sara takes 50 zeds and Menzi takes  $\frac{3}{4}$  of the remaining zeds.

How many zeds are left for Peter?

- A. 10
- B. 40
- C. 50
- D. 60



M02\_02

### Learner performance

	A	B	C	D	Omitted
South Africa	17,3	26,7	26,1	26,9	2,4
International average (n=38)	10,4	49,4	14,8	22,3	2,6

### South African curriculum

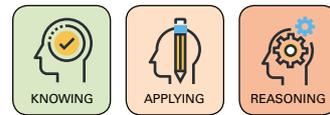
Whole numbers – Calculations with fractions (common fractions and decimal fractions)

### Types of errors

The choice by the learners is almost equally spread between the correct answer of B (26,7%) and incorrect answers of C (26,1%) and D (26,9%). This may be attributed to the possible confusion of zeds taken by Menzi and those remaining for Peter. At least the majority of learners had an idea that the number of zeds for Peter could not be as low as given in option A; however, the closeness of answers seems to suggest some confusion in the process of calculation. The problem in this case does not seem to be with the common fractions.

Learners struggle with applying principles of fractions of whole numbers. They did not read and understand the information provided. Therefore, they struggle to provide the necessary steps to solve the problem and to compare and check if their workings are accurate. The lack of information might result in learners guessing what the correct answer is.

The use of the term zeds as a monetary value may also have caused confusion for learners who are used to working with the term Rand.



### Recommended action

Teachers should teach techniques for answering multiple-choice questions. Emphasis should be placed on the logical flow of construction responses and problem solving. For example, one should always calculate first and then look at the available options.

**Cognitive area: Applying**

**QUESTION 17**

**Flowers Maria sold**

Maria sold 4 types of flowers. She sold an equal amount of tulips and orchids.

A. What percentage of the flowers she sold were tulips?  
Answer: \_\_\_\_\_ %

B. Maria sold 40 carnations. What is the total number of flowers she sold?  
Answer: \_\_\_\_\_

M02\_03

### Learner performance

Question 17A	Correct	Incorrect	Omitted
South Africa	15,0	82,1	2,4
International average (n=38)	46,2	46,3	6,8

Question 17B	Correct	Incorrect	Omitted
South Africa	4,3	89,6	5,3
International average (n=38)	29,9	54,0	15,2

In TIMSS, this question is represented in the data content section.

### South African curriculum

Representation of data (although this seems to be different from the TIMSS content domain of Number).

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Types of errors

The correct response for part A was 22,5 or  $22\frac{1}{2}$ . A very high percentage of learners gave an incorrect response. This may be due to the fact that this question is a combination of a word problem and representation of data. It suggests that most of the learners struggle with both content areas. In this particular question, learners misinterpreted the pie chart and the related data.

The correct response for part B was 200. The percentage of incorrect responses is higher for this part of the question (89,6%). It speaks fundamentally to the type of error raised in the previous question about a combination of a word problem and representation of data. This clearly presents a major problem for most learners. This question proved to be challenging for the learners because it is a ratio question that requires a high level of reasoning. South African learners receive very little training and teaching on how to reason.

### Recommended action

Teachers need to facilitate an understanding of pie charts and methods of interpretation. For example, it should be emphasised that pie charts add up to 100%. The teachers need to focus on integrating reasoning skills into teaching methodologies. While this question could be solved using ratios, it is imperative that learners know how to reason in order to obtain the correct answer.

#### Cognitive area: Applying

#### QUESTION 18



In John's house there are stools stacked together.



One stool is 49 cm high.

When 2 stools are stacked together the stools are 55 cm high.

How high above the ground is the top of a stack of 6 stools?

- A. 79 cm
- B. 85 cm
- C. 110 cm
- D. 165 cm

M03\_02

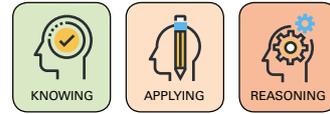
### Learner performance

	A	B	C	D	Omitted
South Africa	24,2	16,5	26,4	31,2	1,6
International average (n=38)	39,5	19,5	15,0	24,3	1,6

### South African curriculum

Numbers, operations and relationships.

Patterns, functions and algebra and Numbers and operations.



### Types of errors

The majority of learners chose incorrect options C (26,4%) and D (31,2%). These options are more than double the height of 2 stools of 55 cm. It may be deduced that the learners relate the increase in the number of stacked chairs to the similar increase in the height of the stacked stools. This may be attributed to the problem of estimation based on real numbers. The higher numbers of 110cm and 165cm seemed more plausible and appealing to the learners than the lower amounts.

Learners are exposed to patterns in the foundation phase. However, they are not exposed to complex patterns of this nature. They battled with this question because they were unfamiliar with the complexity and difficulty of the question.

### Recommended action

Teachers should increase learners' exposure to complex patterns through classroom exercises. Classroom exercises should involve integration of reasoning skills. There needs to be increased integration of content from earlier grade. Action can also be linked to contextual Factor three.

#### Cognitive area: Applying

#### QUESTION 19



You have a recipe for a cake that needs 2 eggs and 0,3 litres of milk.



You want to make as large a cake as you can and you have 5 eggs.

How many litres of milk do you need to make the largest possible cake?

Answer: \_\_\_\_\_ litres

M03\_05

### Learner performance

	Correct	Incorrect	Omitted
South Africa	7,8	89,2	3,0
International average (n=38)	37,7	57,2	4,8

### South African curriculum

Ratio and proportion question: Problem solving.

### Types of errors

The correct response for this question was 0,75 or equivalent. The majority of learners (89,2%) again gave an incorrect response. This is also observed to a high percentage at the international average of 57,2% which indicates that this problem was not easy for learners in many countries. This suggests that learners experience problems in converting word problems into formulas (equations). It is further concerning as the problem is an example of application of mathematics in real-life situations. Learners lack the necessary understanding required to solve this ratio question. More specifically, they struggle with conversion of ratios.

Contextual factors and items  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### Recommended action

Teachers need to increase the exposure of learners to ratio-related exercises. This should include knowledge application and conversion exercises. Word problems relating to real-life situations are an important area for teachers to focus on.

#### Cognitive area: Applying

#### QUESTION 20



M05\_04

Which is larger,  $\frac{7}{12}$  or  $\frac{2}{3}$  ?



Explain your answer.

### Learner performance

	Correct	Incorrect	Omitted
South Africa	7,4	87,3	4,9
International average (n=38)	37,2	55,2	7,3

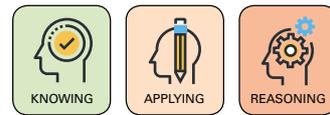
### South African curriculum

Whole numbers – Calculations with common fractions and equivalence.

### Types of errors

Correct responses stated that two-thirds was greater based on a common denominator or on decimals. Incorrect responses were all other responses, including stating that two-thirds was greater but without adequate explanation, for example "2/3 because it gives us 0.666...7". The majority of learners gave an incorrect response, suggesting that most of the learners have difficulty with common fractions, especially fractions with different denominators.

An example of an incorrect answer is provided in Figure 4, where a learner became confused with the task and instead of comparing the fractions, multiplied them. This shows either that the learner is unsure of how to compare fractions or did not understand the phrasing of the question. Other learners also struggled with common principles of fractions and mistook a large number to be higher, stating, "7/12 because if you add the numbers they will not give you same answers; 7/12 will give you the larger number" and "7/12 because they have the biggest number and you can multiply them".



**Figure 4:** Question 20 learner incorrect response example

Which is larger,  $\frac{7}{12}$  or  $\frac{2}{3}$  ?

Explain your answer.

$$\frac{7}{12} \times \frac{2}{3}$$

$$\frac{21 + 24}{12}$$

$$\frac{45}{12}$$

**Recommended action**

Teachers should focus on teaching common fractions with different denominators and their conversion to common denominators. It may also be helpful to introduce conversion to other forms of common fractions or even decimal fractions. Furthermore, they should focus on the equivalence, multiplication and mixed fractions and conversion of mixed numbers in the foundation phase.

*Cognitive area: Reasoning*

**QUESTION 21**  $\frac{2}{3}x + 1$  is a whole number.



Which of these statements about x must be true?

- A. x must be an odd number
- B. x must be an even number
- C. x must be a number greater than 3
- D. x must be a number divisible by 3

M03\_03

**Learner performance**

	A	B	C	D	Omitted
South Africa	23,3	29,3	18,0	25,7	3,7
International average (n=38)	17,5	17,8	15,9	43,8	4,8

**South African curriculum**

Numbers, operations and relationships: Whole numbers.

Patterns, functions and algebra.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Types of errors

Given the spread between option A (23,3%), option B (29,3%) and option D (25,7%), it is not clear what informed the choices of the learners. It is possible that they were struggling with the divisible common fraction part, without taking into account that the outcome must actually be a whole number. It is also possible that the learners were guessing. Furthermore, learners do not understand that  $x$  belongs to the numerator and not the denominator.

### Recommended action

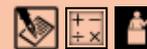
Teachers must provide learners with examples that incorporate equations and multiple-choice options of answers where learners are required to identify which statement must be true. Teachers must emphasise that  $x$  belongs to the numerator and not the denominator.

#### Cognitive area: Reasoning

#### QUESTION 22



George and Temba bought identical hockey sticks from different shops.



The usual price of the hockey sticks had been the same in both shops.

George bought a hockey stick for 20% less than the usual price.

Temba paid  $\frac{3}{4}$  of the usual price for his hockey stick.

Who paid less for his hockey stick?

M/06\_02

### Learner performance

	Correct	Incorrect	Omitted
South Africa	6,4	89,7	3,9
International average (n=38)	29,0	60,9	10,0

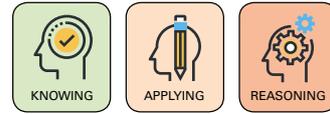
### South African curriculum

Numbers, operations and relationships: Whole numbers.

Patterns, functions and algebra.

### Types of errors

Approximately 90% of learners answered this question incorrectly. Word problems, particularly those which are information-laden, are confusing to South African learners. In additions, learners have difficulty with fractions, especially when conversions from fractions to percentages, and vice versa, are required.



### Recommended action

Classroom exercises should include complex word problems, which include multiple levels of information. Furthermore, learners should be taught how to sift through the required information in order to identify the relevant variables to be used for calculations. A focus on fractions, and conversions from fractions to percentages, and vice versa, is also required.

#### Cognitive area: Reasoning

#### QUESTION 23



On a TV game show, each correct answer is worth 2 points and 1 point is deducted for each incorrect answer. Bongani, Camilla, and Luca have 11 points each as a final score. Each has given the same number of correct and incorrect answers.



Complete the following table.

	Correct Answers	Incorrect Answers	Final Score
Bongani		5	11
Camilla	7		11
Luca			11

M06\_03

### Learner performance

	Correct	Partially correct	Incorrect	Omitted
South Africa	3,5	1,4	90,6	4,3
International average (n=38)	24,8	11,9	55,9	7,2

### South African curriculum

Equation, modelling patterns.

### Types of errors

This high percentage of incorrect responses (90,6%) again highlights that South African learners struggle with fractions. This type of question is a higher-level question that is not included in the intermediate and senior phase curriculum. Therefore, South African learners might be unfamiliar with this type of question and reasoning.

### Recommended action

Because this is a higher-level question, more information should be provided in the question so that learners can attempt to use the required reasoning skills. Teachers should include similar examples in teaching in order to equip learners with the skills that will allow them to answer questions such as this correctly.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Cognitive area: Reasoning

#### QUESTION 24



Tom and his brother Peter received the same amount of money.



Tom spent  $\frac{1}{3}$  of his money on books. He spent  $\frac{3}{5}$  of the remaining money to buy a new pair of shoes.

Peter spent  $\frac{3}{5}$  of his money to buy a new pair of shoes.

Who spent more for shoes?

(Check one box.)

- Tom spent more money for shoes
- Peter spent more money for shoes
- They both spent the same amount for shoes

Explain your answer.

M06\_05

### Learner performance

	Correct	Incorrect	Omitted
South Africa	4,0	93,4	2,4
International average (n=38)	20,0	73,4	5,0

### South African curriculum

Numbers, operations and relationships: Whole numbers.

Patterns, functions and algebra

### Types of errors

Ninety-three percent of South African learners were unable to answer this question correctly. Learners struggle with information-laden questions and word problems that involve calculations with fractions. The incorporation of fractions with different denominators in the questions would have added more complexity which may have confused learners further. Learners find it difficult to apply fractions to real-life word problems.

### Recommended action

Classroom exercises should include complex word problems, which include multiple levels of information. Furthermore, learners should be taught how to sift through the required information in order to identify the relevant variables to be used for calculations. A focus on fractions with different denominators is also required. Emphasis should be placed on the application of common fractions and related conversions to real-life problems.



## TIMSS content area: Algebra

*Cognitive area: Knowing*

**QUESTION 25**

$x + y = 5$

What is the value of  $3x + 3y$ ?

A. 5  
B. 8  
C. 15  
D. 45

M01\_03

### Learner performance

	A	B	C	D	Omitted
South Africa	25,9	18,4	45,1	8,1	2,4
International average (n=38)	11,0	13,4	63,2	8,7	3,6

### South African curriculum

Patterns, functions and algebra: Equations.

Covered in the third term in the South African Grade 9 CAPS curriculum.

### Types of errors

Forty-five percent of South African Grade 9 learners answered this question correctly. The high percentage of correct responses might be due to the fact that this content is covered in the third term in the South African Grade 9 CAPS curriculum. South African learners, however, lack the basic knowledge required to balance equations. Most learners failed to identify that they were supposed to multiply by 3. Twenty-six percent of learners answered option A, indicating they may have assumed that the second equation would equal the first, and not gone beyond that to identify what was required in the question.

### Recommended action

Remedial action can be linked to contextual Factor two (Back to basics): Laying the foundation of equations, especially the balancing of equations; integration of algebra and graphs.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 26** 

Which of the expressions below represents one fifth of  $x$ ?

A.  $x + \frac{1}{5}$   
 B.  $\frac{5}{x}$   
 C.  $x - \frac{1}{5}$   
 D.  $\frac{x}{5}$

M01\_05

### Learner performance

	A	B	C	D	Omitted
South Africa	33,3	22,8	13,4	28,8	1,6
International average (n=38)	19,8	20,5	9,2	48,3	2,0

### South African curriculum

Patterns, functions, algebra: Fractions.

### Types of errors

Approximately a third of learners answered incorrectly with option A. These learners may have identified the two variables in the question as being present in this option, or they may have misunderstood the question as requiring an addition calculation. Learners may have had difficulty combining whole numbers with fractions.

### Recommended action

Classroom exercises should focus on the basic foundations of fractions, working with a combination of whole numbers and fractions, and understanding of mathematical terminology.

**Cognitive area: Knowing**

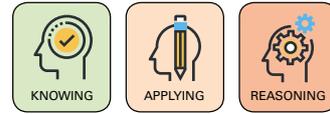
**QUESTION 27** 

Which pair of numbers  $(x, y)$  makes the equation  $3x + 4y = 24$  true?

A.  $(0, 8)$   
 B.  $(3, 4)$   
 C.  $(4, 3)$   
 D.  $(6, 0)$

M01\_07



### Learner performance

	A	B	C	D	Omitted
South Africa	5,0	46,2	40,4	6,1	1,9
International average (n=38)	6,2	23,8	61,1	5,4	2,9

### South African curriculum

Graphs.

### Types of errors

Forty percent of South African learners answered this question correctly. This could be due to the fact that this content is covered in the third term of the Grade 9 CAPS curriculum. However, learners still lack appropriate knowledge and understanding of substitution, especially in this format or notation. The notation used in this question is a higher-order question that is not included in the Grade 9 curriculum/textbook. Lack of understanding creates a bigger gap for the senior phases. Forty-six percent of learners answered with option B, which was (3, 4). These are the numbers which appear in the equation. Learners may therefore have assumed that these were the correct numbers based on what was provided in the question.

### Recommended action

Classroom teaching should emphasise substitution techniques and various types of graphs. With regard to the notation, the South African content uses a semicolon and not a comma. The use of a semicolon might be helpful in increasing the learners' understanding of the question. Integration and threading the various topics in the curriculum across phases is also relevant.

Cognitive area: Knowing

**QUESTION 28**



Complete the table of values for  $y = 2x^2 - 3$

x	-2	1	4
y			




M02\_05

### Learner performance

	Correct	Incorrect	Omitted
South Africa	17,3	74,4	5,4
International average (n=38)	18,0	57,2	23,5

### South African curriculum

Algebra: Equations.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Types of errors

Almost three-quarters of learners were unable to answer this question correctly. This highlights difficulties with working with equations, and a lack of understanding of substitution. Learners may also have had trouble with working out multiple substitutions for the equation.

### Recommended action

Senior phase teachers almost always have to reteach the basic knowledge of substitution. In order to remedy this problem, Intermediate phase teachers need to intensify their teaching of basic concepts. Furthermore, the teaching needs to include complex examples that test the depth of knowledge and understanding of the learners.

*Cognitive area: Knowing*

**QUESTION 29**



$$\frac{a^2}{2} - 6a + 36$$

Which is the value of this expression when  $a = 3$ ?

- A. 58,5
- B. 27
- C. 22,5
- D. 21



M03\_06

### Learner performance

	A	B	C	D	Omitted
South Africa	12,8	31,3	32,1	22,0	1,7
International average (n=38)	14,3	20,1	49,1	12,4	3,8

### South African curriculum

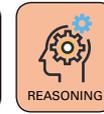
Algebra: Equations.

### Types of errors

Almost a third of learners answered this question correctly. Learners however lack the necessary understanding of substitution, and have difficulty working with whole numbers and fractions.

### Recommended action

Senior phase teachers almost always have to reteach the basic knowledge of substitution. In order to remedy this problem, intermediate phase teachers need to intensify their teaching of basic concepts. Furthermore, teachers should provide examples which incorporate working with a combination of whole numbers and fractions.



**Cognitive area: Knowing**

**QUESTION 30**

$$y = \sqrt{x - 9}$$



Which of these is a value of y when x = 25?

- A. 3
- B. 4
- C. 8
- D. 16

M03\_06

**Learner performance**

	A	B	C	D	Omitted	Girl % right	Boy % right
South Africa	24,9	36,2	10,5	26,6	1,3	37,7	34,5
International average (n=38)	8,9	53,5	11,3	24,0	1,9	55,1	51,9

**South African curriculum**

Patterns, functions and algebra

**Types of errors**

It appears that learners have misconceptions about the principles of division and square roots. Over a quarter of learners answered incorrectly with option D. These learners appear to have calculated the right side of the equation (25-9), giving an answer of 16, without completing the calculation using the square root. These learners may not have understood how to calculate the square root or may not have read the question carefully. A further quarter of learners answered with option A (3). These learners may have miscalculated the square root of 16 or may simply have calculated the square root of the 9 given in the equation.

**Recommended action**

Teachers should include various examples of substitution and square roots, particularly in the context of equations. Teaching needs to include complex examples that test the depth of knowledge and understanding of the learners.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 31**  

$4x = 7$

 What is the value of x?

A. 28  
B. 11  
C.  $1\frac{3}{4}$   
D.  $\frac{4}{7}$

M05\_06

### Learner performance

	A	B	C	D	Omitted
South Africa	22,4	19,5	28,7	26,6	1,6
International average (n=38)	15,3	8,7	51,1	21,6	2,9

### South African curriculum

Patterns, functions and algebra: Fractions.

### Types of errors

South African learners lack conceptual understanding of inversion operations and the equality of equations. The most common incorrect answer was option D.  $\frac{4}{7}$ . These learners seem to have chosen the answer which contained the two numbers provided in the question, without understanding what was required. The next most common incorrect answer was option A (28). Learners who gave this answer appear to have multiplied the two numbers (4 and 7) which were given in the question.

### Recommended action

Teachers should focus on emphasising conceptual understanding of inverse operations and on examples of inverse operations. They should also focus on substitution in equations and determining the value of x, particularly in equations where the answer includes a whole number and a fraction.



**Cognitive area: Knowing**

**QUESTION 32**



Which equation is satisfied by  $x = 2$  and  $y = 7$ ?



- A.  $7x - 2y = 0$
- B.  $2x - 7y = 0$
- C.  $7x + 2y = 14$
- D.  $2x + 7y = 9$

M05\_08

**Learner performance**

	A	B	C	D	Omitted
South Africa	17,8	27,3	20,6	31,8	1,9
International average (n=38)	44,4	14,9	14,8	21,8	3,6

**South African curriculum**

Patterns, functions and algebra: Equations.

**Types of errors**

The most common incorrect answer given by South African learners was option D. These learners appear to have associated the variables given together in the question, therefore choosing an option with  $2x$  and  $7y$ . Additionally, the right side of the equation (9) is equal to the addition of the numbers on the left side of the equation (2 and 7). The second most commonly given incorrect answer was option B. These learners again appear to have associated the variables in the question with each other when choosing an answer. This question highlights learners' difficulty with substitution and solving equations.

**Recommended action**

Classroom teaching should focus on equations and substitution, in order to ensure that learners understand how to work with variables such as  $x$  and  $y$  in equations. Teachers should also incorporate test and exam answering techniques into their teaching, especially techniques related to multiple-choice questions. Learners need to be taught to read questions carefully and take into account all of the information provided when answering questions.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 33**  

  $a = 1 + x$  and  $b = 1 - x$ .

A. What is  $a + b$ ?

Answer: \_\_\_\_\_

B. What is  $a - b$ ?

Answer: \_\_\_\_\_

M05\_11

### Learner performance

Question 33A	Correct	Incorrect	Omitted
South Africa	15,1	80,6	3,2
International average (n=38)	37,3	47,0	14,6

Question 33B	Correct	Incorrect	Omitted
South Africa	1,4	93,4	4,0
International average (n=38)	13,9	68,2	16,8

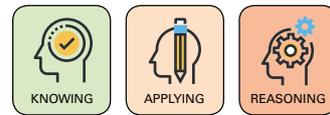
### South African curriculum

Patterns, functions and algebra: Substitution.

Patterns, functions and algebra: Equations.

### Types of errors

Incorrect responses for part A were either  $2 + 2x$  or any other incorrect answer. There were a variety of incorrect equations provided (an example is indicated below), showing that learners struggled with understanding this item. This may be because they are unable to equate the equations and instead become overwhelmed with creating complex equations. One learner restated the question but in words rather than algebraic terms, i.e. "variables adding," showing that they do not understand this type of question. This may be linked to language. See Figure 5 as an example of common incorrect responses.



**Figure 5:** Question 32 learner incorrect response example

$a = 1 + x$  and  $b = 1 - x$ .

A. What is  $a + b$ ?

Answer:  $ab = \frac{1+x}{1-x^2} \cdot 1-x$

B. What is  $a - b$ ?

Answer:  $1-x$

For part B, the high percentage of incorrect responses (93,4%) indicates that South African learners battle with substitution and distribution laws.

### Recommended action

Teachers should include complex examples of distribution laws through drilling exercises. Emphasis on mastering equations is required.

**Cognitive area: Knowing**

**QUESTION 34**

$a = 5$  and  $b = 2$ .

What is the value of  $a^2b - 3(a - b)$ ?

Answer: \_\_\_\_\_

M07\_07

### Learner performance

	Correct	Incorrect	Omitted
South Africa	10,5	81,6	7,8
International average (n=38)	31,5	55,6	12,5

### South African curriculum

Patterns, functions and algebra: Equations.

### Types of errors

The correct response was 41. The majority of learners again struggled with this type of question, highlighting their lack of understanding of working with equations and substitution.

### Recommended action

Classroom teaching should focus on equations and substitution, in order to ensure that learners understand how to work with variables such as  $a$  and  $b$  in equations. Examples of complex equations should be incorporated, and the application of BODMAS should be reiterated.

Contextual factors and icons  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### Cognitive area: Applying

#### QUESTION 35



A class visits a museum. Lunch for the whole class costs B zeds. It costs 4 zeds per student to pay for entry to the museum. There are p students in the class.



The total cost for the trip is K zeds. What is the formula for K?

- A.  $K = B + 4$
- B.  $K = B + 4p$
- C.  $K = B + p$
- D.  $K = (B + p) \times 4$

M01\_08

### Learner performance

	A	B	C	D	Omitted
South Africa	11,3	32,3	8,0	46,7	1,1
International average (n=38)	7,1	53,1	7,1	29,9	2,1

### South African curriculum

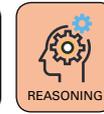
Patterns, functions and algebra: Equations.

### Types of errors

Approximately a third of South African learners answered this question correctly. The majority of learners selected incorrect option D. These learners selected an answer which contained all of the relevant variables. They may, however, have misunderstood that B related to the whole class, while the entry to the museum (which cost 4 zeds) related to individual students. They therefore multiplied both of these variables by 4, rather than only the number of students. The question highlights learners' difficulty with the application of equations and variables in real-life word problems.

### Recommended action

Teachers have to emphasise substitution of variables in the solution of word problems, and provide learners with examples of word problems that incorporate real-life situations. The use of formulas for solving word problems also requires attention.



**Cognitive area: Applying**

**QUESTION 36**



$x > 3$  and  $y < 2$ .



Which values of  $x$  and  $y$  satisfy this condition?

- A.  $x = 2, y = -1$
- B.  $x = 3, y = 2$
- C.  $x = 4, y = 2$
- D.  $x = 5, y = -1$

M01\_09

**Learner performance**

	A	B	C	D	Omitted
South Africa	11,3	60,2	9,2	16,6	2,2
International average (n=38)	10,7	23,7	9,4	52,5	2,8

**South African curriculum**

Patterns, functions and algebra: Inequalities

**Types of errors**

The high percentage of incorrect responses shows that learners do not understand rational and natural numbers. They also display very little understanding of mathematical signs. As noted earlier, South African learners do not read a question with understanding. Sixty percent of learners answered incorrectly with option B. This once again highlights that when learners are unsure how to answer a question, they choose the answer which contains the same numbers as those provided in the question. Additionally, from the information in the question, these learners associated  $x$  with 3, and  $y$  with 2.

**Recommended action**

Classroom teaching should be focused on emphasising the principles of mathematical terms such as “and” as well as “or”; mathematical signs, as well as the interpretation of a question. Furthermore, there needs to be increased exposure to rational numbers and natural numbers in the Grade 7 curriculum.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 37**



$$t = x - \frac{6,5}{1\,000} y$$


The formula above gives the temperature  $t$  °C at a place  $y$  metres above sea level when the temperature at sea level is  $x$  °C. What is the temperature at the top of a mountain 2 000 m high when the temperature at sea level is 21 °C?

Answer: \_\_\_\_\_ °C

M01\_10

### Learner performance

	Correct	Incorrect	Omitted
South Africa	2,9	85,7	10,6
International average (n=38)	16,2	54,2	28,7

### South African curriculum

Algebra examples: Problem solving.

### Types of errors

The correct answer for this question was 8. As shown in previous questions, South African learners struggle with reading, understanding, and interpretation of questions, as well as the identification of key information. The question combines equations, substitution and fractions, resulting in a high level of complexity which learners struggled with. The learners may also have been unfamiliar with substituting two variables as this is only introduced in the last quarter of Grade 9.

### Recommended action

Classroom teaching needs to incorporate complex questions which include a range of elements, such as substitution, fractions and word problems. Learners struggle with complex questions, and require practical examples to develop the necessary skills to answer these questions. Teachers should also highlight questions which focus on real-world situations.



**Cognitive area: Applying**

**QUESTION 38**

Find the values of  $x$  and  $y$  such that both equations are true.



$$3x + y = 13$$

$$5x - y = 27$$

M02\_04

**Learner performance**

	Correct	Incorrect	Omitted
South Africa	1,4	86,8	11,0
International average (n=38)	17,9	56,9	24,2

**South African curriculum**

This type of question is out of the scope of the CAPS Grade 9 curriculum.

**Types of errors**

As in the case above, South African learners struggle with reading, with understanding, interpretation of the question and identification of key information. Learners also have difficulty with substitution and solving equations. The learners might also be unfamiliar with substituting two variables as this is only introduced in the last quarter of Grade 9.

**Recommended action**

Teachers need to give learners examples in which substitution of variables in equations is emphasised. Specific attention is required for questions where two equations must be true, as this adds complexity and will confuse learners, particularly those who are struggling with the foundational concepts.

Contextual factors and icons  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### Cognitive area: Applying

#### QUESTION 39



Nosisi described the graph of a function:



- The graph is a straight line.
- The graph intercepts the y-axis at 3.

Which could be the function of Nosisi's graph?

- $y = x^2 + 3$
- $y = 3x + 1$
- $y = 3x^2 - 1$
- $y = x + 3$

M02\_06

### Learner performance

	A	B	C	D	Omitted
South Africa	13,5	23,0	15,6	42,6	3,0
International average (n=38)	10,1	23,8	15,2	41,9	7,5

### South African curriculum

Patterns and functions: Graphs.

### Types of errors

Approximately 43% of South African learners answered this question correctly. This was just above the international average. The types of errors committed in this question can be linked to the lack of basic understanding of straight line graph, such as misunderstanding the y-intercept and the x intercept of a linear graph.

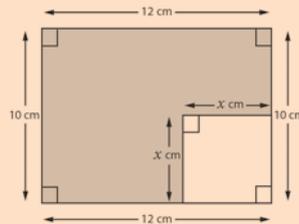
### Recommended action

Classroom teaching should go back to the basic understanding of straight line graphs, especially the inversion of functions and equations, for example, the difference between  $y = 2x + 1$  and  $y = x^2 + 1$  and  $y^2 = x + 1$ .



Cognitive area: Applying

QUESTION 40



Write an expression in terms of  $x$  for the area of the **shaded** portion of the figure.

Answer: \_\_\_\_\_ cm<sup>2</sup>

M08\_07

Learner performance

	Correct	Incorrect	Omitted
South Africa	0,8	94,5	4,5
International average (n=38)	16,2	68,7	14,7

South African curriculum

Patterns and functions.

Types of errors

The correct answer for this question was  $120 - x^2$  or equivalent. Approximately 95% of learners answered this question incorrectly. Learners did not seem to understand the question and what was expected of them. They did not understand that they needed to carry out the steps for calculating the area of the shaded part of the figure. It is highly probable that this was never taught in the classroom setting.

Recommended action

Classroom teaching of the area of shapes should start with numbers and progress to complex examples of shapes and functions. Teachers should ensure that learners are given examples which incorporate different types of variables, and require learners to provide relevant expressions.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 41**




The shape above is a rectangle, with length  $l$ , and width  $w$ .

If the length is doubled and the width stays the same, which formula gives the area ( $A$ ) of the new rectangle?

A.  $A = 2l + 2w$   
 B.  $A = 2l + 4w$   
 C.  $A = 2lw$   
 D.  $A = 4lw$

M03\_09

### Learner performance

	A	B	C	D	Omitted
South Africa	25,0	26,6	29,5	16,7	1,6
International average (n=38)	18,8	17,4	50,7	10,3	2,3

### South African curriculum

Measurement

### Types of errors

South African learners battle with understanding the conceptual difference between perimeter and area. This might have been further exacerbated by the use of unfamiliar language or a term such as “width” instead of the familiar term of “length.”

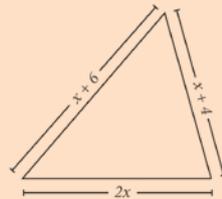
### Recommended action

Classroom teaching should increase exposure to different mathematical terms with the same or similar meaning. This can be done through practical and drilling exercises. These exercises should also place emphasis on the understanding of formulas.



Cognitive area: Applying

QUESTION 42



The sum of the lengths of the sides of the triangles in 30cm.

Write an equation that would enable you to find the value of x.

M05\_10A

Learner performance (%)

	Correct	Incorrect	Omitted
South Africa	4,9	84,6	9,8
International average (n=38)	28,7	48,8	21,7

South African curriculum

Measurement and Algebra.

Types of errors

The correct response is  $4x + 10 = 30$  or the equivalent (e.g.,  $4x = 30 - 10$ ). As a large majority of learners (84,6%) answered this question incorrectly, South African learners did not possess the required basic knowledge of the properties of a triangle as well as calculating the perimeter of a triangle.

Recommended action

Classroom teaching should increase exposure to different mathematical terms with the same or similar meaning. This can be done through practical and drilling exercises. These exercises should also place emphasis on the understanding of formulas.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 43** Which is a correct step in solving the equation   

  $4x - 3 = 2x - 7$ ?

A.  $4x - 2x = 3 - 7$   
 B.  $4x + 2x = 3 - 7$   
 C.  $4x - 2x = 7 - 3$   
 D.  $4x + 2x = 7 - 3$

M06\_06

### Learner performance

	A	B	C	D	Omitted
South Africa	30,0	22,5	26,8	18,1	2,2
International average (n=38)	43,3	17,9	23,4	11,8	2,8

### South African curriculum

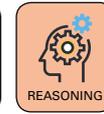
Patterns, functions and equations.

### Types of errors

South African learners struggled with balancing equations, the understanding of integers and associated signs. This can be associated with an inadequate teaching foundation of numbers, equations and distribution.

### Recommended action

Appropriate remedial action would require going back to basics in the classroom setting. In the classroom, teachers can enforce the understanding of balancing equations and distribution laws through drilling exercises.



### Cognitive area: Applying

#### QUESTION 44



Mpho and Tom went to town to the same shop to buy some books and pens.



Mpho bought 5 books and 2 pens and paid 74 zeds.

Tom bought 1 pen and 3 books and paid 42 zeds.

Which pair of equations could represent this situation?

A.  $5x + 2x = 74$   
 $y + 3y = 42$

C.  $5x + 2y = 74$   
 $3x + y = 42$

B.  $5x + 2y = 74$   
 $x + 3y = 42$

D.  $5y + 2y = 74$   
 $3x + y = 42$

M06\_07

#### Learner performance

	A	B	C	D	Omitted
South Africa	25,4	33,9	28,3	9,4	2,2
International average (n=38)	20,7	27,4	42,4	5,7	3,0

#### South African curriculum

Algebra: Equations.

#### Types of errors

The correct answer to this question is option C. This question required the learners to interpret simultaneous equations. It required a high level of reading skills, which South African students lack. South African learners tend to do better in clearer and straightforward questions that do not require intensive reading, interpretation and problem solving. The largest proportion of learners (33,9%) chose option B. These learners may have just identified the numbers in the question and then found the equation that followed that order (5, 2, 74 and 1, 3, 42). They did not show understanding that the book and pens respectively must each have the same variable, either x or y.

#### Recommended action

Teachers need to use various resources to familiarise learners with different examples on how principles can be applied. The curriculum should encourage the use of multiple textbooks.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

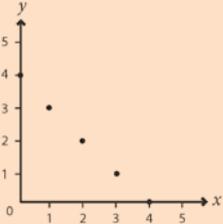
Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 45**

x and y are whole numbers.

What is the relationship between x and y for the points shown on the graph?

A.  $y = x - 4$   
 B.  $x = y - 4$   
 C.  $x + 4y = 4$   
 D.  $x + y = 4$

M07\_05

### Learner performance

	A	B	C	D	Omitted
South Africa	16,6	12,4	24,2	44,4	2,1
International average (n=38)	14,2	15,1	18,0	47,0	5,3

### South African curriculum

Graphs.

### Types of errors

Although South African learners performed fairly well in this item – just below the international average – learners battle with understanding techniques required to make  $y$  the subject of the formula, as well as checking the gradient and the  $y$  intercept.

### Recommended action

Learners have to be taught on the components of an equation such as gradients and intercepts. Classroom exercises should include more examples of functions and graphs, as well as associated shifts to the right and the left.



**Cognitive area: Applying**

**QUESTION 46**



A car travels at an average speed of 50 km/h.

Which formula represents the distance in kilometres,  $d$ , that the car travels in  $t$  hours?

- A.  $d = 50 - t$
- B.  $d = \frac{t}{50}$
- C.  $d = \frac{50}{t}$
- D.  $d = 50t$

M07\_06

**Learner performance**

	A	B	C	D	Omitted
South Africa	9,8	31,4	34,8	22,5	1,3
International average (n=38)	7,0	22,1	31,2	36,3	3,0

**South African curriculum**

Algebra: Equations.

The Department of Basic Education’s team of curriculum experts placed this question within the knowledge cognitive domain because it is inconsistent with the rest of the application questions. It appears to be a combination of knowledge and applying.

**Types of errors**

The correct answer is D. Most learners chose either option B or C. This question required learners firstly, to know the formula for calculating distance, and then to apply the formula to the question. South African learners lack the required knowledge of the calculation of distance.

**Recommended action**

Intermediate phase teachers need to use drilling exercises and memory techniques so that learners know what formulas to use for specific questions.

Contextual factors and items  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Reasoning**

**QUESTION 47**      -3, 6, -12, 24, ...  

 Write a rule so that you know if you know any term in this pattern you can find the next term.

M05\_05

### Learner performance

	Correct	Incorrect	Omitted
South Africa	8,0	81,9	9,6
International average (n=38)	25,2	52,8	21,7

### South African curriculum

Patterns, functions and algebra: Numeric and geometric patterns.

### Types of errors

The correct responses included: Multiply each term by  $-2$  to get the next term/multiply by  $2$  and change the sign/or equivalent. The majority (81,9%) of South African learners failed to identify the rule. This might be due to the fact that they are unfamiliar with relationships between numbers, especially when negative numbers are concerned.

### Recommended action

This question is not in line with the Grade 9 South African curriculum. However, learners who have developed their reasoning skills should be able to answer this question.

**Cognitive area: Reasoning**

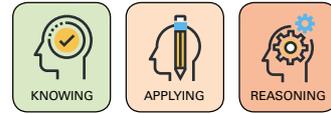
**QUESTION 48**      Write in the missing term in this sequence:    

 1, 1, 2, 3, 5, 8, 13, \_\_\_\_, 34, 55.

M05\_07

### Learner performance

	Correct	Incorrect	Omitted
South Africa	30,1	64,3	5,2
International average (n=38)	45,1	43,4	11,1



### South African curriculum

Patterns, functions and algebra: Fibonacci sequence.

### Types of errors

The correct response is 21 (each number is the addition of the two previous numbers in the pattern). South African learners were unable to identify the Fibonacci sequence. It is most likely that the sequence was never taught to most learners. It requires high-level reasoning, which South African learners generally lack.

### Recommended action

Teachers should teach various types of patterns, including the Fibonacci sequence.

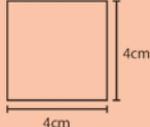
**Cognitive area: Reasoning**

**QUESTION 49** Jay makes a pattern of squares. He increases the length of the side of a square by the same amount each time.

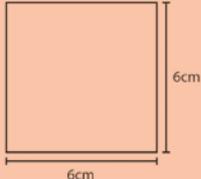
Here are the first 3 squares in the pattern.



1st Square



2nd Square



3rd Square

A. What will be the area of the 5th square?

A. 100 cm<sup>2</sup>  
B. 64 cm<sup>2</sup>  
C. 25 cm<sup>2</sup>  
D. 10 cm<sup>2</sup>

B. What will be the area of the 9th square?

M06\_08A

### Learner performance

Part A	A	B	C	D	Omitted
South Africa	17,9	12,2	14,8	51,5	2,4
International average (n=38)	48,2	17,7	7,0	22,7	3,3

Part B	Correct	Partially correct	Incorrect	Omitted
South Africa	0,5	1,7	89,8	7,8
International average (n=38)	9,2	8,9	78,0	19,5

### South African curriculum

Patterns and measurement.

Contextual factors and items  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### Types of errors

South African learners struggle to read with understanding and to interpret information. As a result, learners missed the calculation of the area component of the question. Learners were first required to calculate the length of the sides of the square and then calculate the area of that square. Most learners chose option D, 10 cm<sup>2</sup>; this may be because they worked out that each side would be 10 cm long, however they did not use this information to calculate the area.

The correct responses for part B of the question are  $4n^2$  OR  $2n \times 2n$  OR  $(2n)^2$ . Learners failed to identify the correct pattern in the previous question (part A) which informed the incorrect responses in part B.

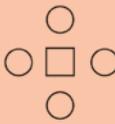
### Recommended action

The reading skills of South African learners need to improve. Drilling exercises that include complex word problems and interpretation of these problems will be helpful in imparting these skills. Problems that include multiple steps and multiple formulas should also be practised.

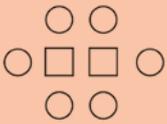
**Cognitive area: Reasoning**

**QUESTION 50** 

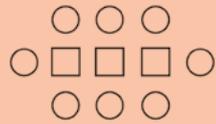
Pattern 1



Pattern 2



Pattern 3



**A. Fill in the number of circles for Pattern 4 and 30 in the table.**

	Pattern 1	Pattern 2	Pattern 3	Pattern 4	→	Pattern 30
Number of squares	1	2	3	4	→	30
Number of circles	4	6	8		→	

**B. Explain how you found the number of circles in pattern 30.**

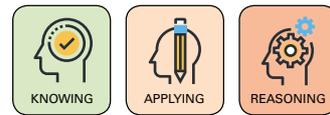
**C. Write a rule for finding the number of circles in pattern n.**

M07\_08A

### Learner performance

Part A	Correct	Incorrect	Omitted
South Africa	23,2	71,5	5,1
International average (n=38)	35,7	55,1	8,6

Part B	Correct	Partially correct	Incorrect	Omitted
South Africa	8,7	9,4	72,8	8,3
International average (n=38)	15,7	16,1	48,9	18,7



Part C	Correct	Partially correct	Incorrect	Omitted
South Africa	8,4	0,4	79,6	1,8
International average (n=38)	19,3	1,6	48,9	29,5

### South African curriculum

Patterns, functions and algebra: Rules.

### Types of errors

The correct response for part A is 62 (scored for pattern 30 only). South African learners failed to identify the rule.

Part B required learners to explain the procedure they followed in order to arrive at the answer i.e.: Explanation of successful method based on doubling and adding 2 or equivalent. This means they had to explain the rule in words instead of formulaically. South African learners struggle to express their thoughts in words or phrases. This results in the lower scores for answers that require the use of phrases or words.

The correct response for part C is  $2n + 2$ . A similar proportion of learners who answered part B correctly also answered this part correctly, exhibiting sound knowledge of number patterns, even when presented in abstract form.

## TIMSS content area: Geometry

*Cognitive area: Knowing*

**QUESTION 51**

In the diagram above, line AB is parallel to line CD. Two right angles are marked.

Which statement is true about the lines l and m?

- Line l is parallel to line m and the same length as line m.
- Line l is perpendicular to line m and the same length as line m.
- Line l is parallel to line m and shorter than line m.
- Line l is perpendicular to line m and shorter than line m.

M05\_12

### Learner performance

Part A	A	B	C	D	Omitted
South Africa	51,0	30,8	10,5	4,2	2,4
International average (n=38)	63,4	21,3	7,3	2,7	4,1

## Item-by-item analysis (continued)

### South African curriculum

Space and shape

### Types of errors

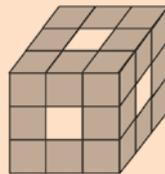
South African learners lack basic knowledge of the properties of parallel lines and right angles. Just over 30% of learners chose option B, this illustrates that they did not understand the term “perpendicular.”

### Recommended action

Classroom teaching needs to emphasise the properties of the rectangle and parallel lines.

#### Cognitive area: Applying

#### QUESTION 52



A cube had 27 small, grey cubes. First, the small cube at the centre of each face was removed. Then, the small cube in the centre was removed. How many cubes were left in the solid?

- A. 4
- B. 16
- C. 20
- D. 24

M01\_11

### Learner performance

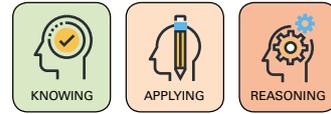
	A	B	C	D	Omitted
South Africa	14,6	9,7	18,9	53,6	1,9
International average (n=38)	7,2	12,5	50,7	26,6	1,9

### South African curriculum

Space and shape: Geometry of 3-D objects.

### Types of errors

Learners are unfamiliar with 3-D representation, and were unable to identify the cube in the centre or “inside” the larger cube. The majority of learners chose option D; these learners may have subtracted the 3 white blocks in the diagram from 27 to arrive at 24.



### Recommended action

The teaching of 3-D representation within the classroom should include breaking down the cube so that learners can understand that a cube isn't just a square. It contains 7 faces, the 7th face is located inside or at the centre of the cube.

**Cognitive area: Applying**

**QUESTION 53** Draw the reflection of the shaded object over the line.

M02\_07

### Learner performance

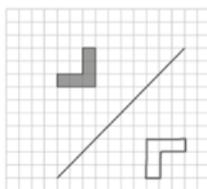
	Correct	Incorrect	Omitted
South Africa	9,8	76,4	11,4
International average (n=38)	42,9	46,7	8,6

### South African curriculum

Space and shapes.

### Types of errors

The correct response is presented below.



South African learners are only exposed to diagonal symmetry in Grade 10 and transformations are only covered in Term 4 of the Grade 9 CAPS curriculum. The diagonal representation of the line of symmetry at the Grade 9 level seems to have confused the learners. The low scores can also be attributed to the fact that teachers had not covered this section of the content before the TIMSS assessment.

### Recommended action

It is recommended that this type of question be excluded from the test as it is beyond the scope of the Grade 9 curriculum.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

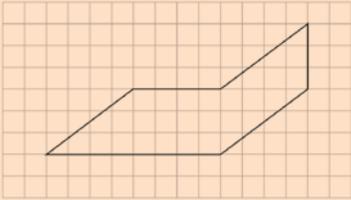
## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 54**



Kethabile is in the fourth grade and knows the formula for finding the area of a rectangle, but no other area formulas. Show Kethabile how she can use the formula for the area of a rectangle to find the area of the shape below. You may mark the shape to help you explain.



M02\_10

### Learner performance

	Correct	Partially correct	Incorrect	Omitted
South Africa	0,1	0,5	76,4	19,2
International average (n=38)	8,2	8,9	51,2	29,0

### South African curriculum

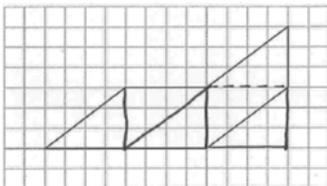
Space and shapes: Measurement.

### Types of errors

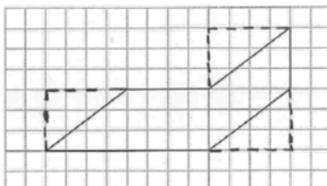
The correct answers for this question show how the figure can be rearranged into one or more rectangles or how the figure can be rearranged into rectangles and triangles and shows that a triangle is half a rectangle.

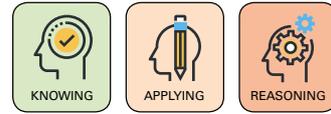
#### Examples:

- *With an explanation:*

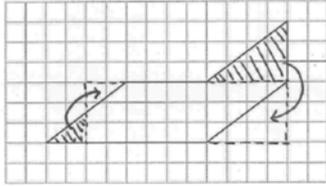


- *With an explanation:*





- *Without an explanation:*



Partially correct answers show how the figure can be divided or arranged into rectangles and triangles, but do not show how to find the area using rectangles. Other partially correct answers show that the triangle is half the rectangle, but do not show that it is half the parallelogram.

This is a high-level operation question. Considering the language barriers South African students face, learners might have thought this question was unclear and lacked specificity. This may account for the high proportion of learners who did not attempt the question.

### Recommended action

A clear and concise phrasing of the question might be helpful in getting learners to understand what is required of them. In the classroom, teachers should focus on the building and progression of formulas.

**Cognitive area: Applying**

**QUESTION 55**

Lines PQ and BC are parallel. What is the value of x?

Answer: \_\_\_\_\_

M03\_12

### Learner performance

	Correct	Incorrect	Omitted
South Africa	21,9	75,2	2,1
International average (n=38)	42,6	48,1	8,5

### South African curriculum

Spaces and shapes: Angles.

Contextual factors and items  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### Types of errors

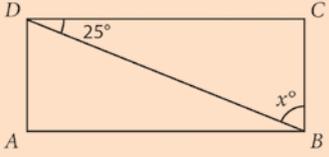
The correct answer is 40°. Learners failed to identify properties of alternate angles and parallel lines. It might be that they lack the basic knowledge of general geometry principles. Therefore, they find difficulty in applying the principles correctly.

### Recommended action

Increase the number of classroom exercises on geometric principles. More marks need to be allocated to this question given the number of steps required in the solution.

**Cognitive area: Applying**

**QUESTION 56** ABCD is a rectangle. What is the value of  $x$ ? 

A. 25  
B. 45  
C. 65  
D. 75

M05\_13

### Learner performance

	A	B	C	D	Omitted
South Africa	38,3	24,0	24,7	10,7	1,3
International average (n=38)	16,0	18,7	54,4	7,7	1,9

### South African curriculum

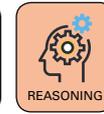
Geometry: Shapes and space.

### Types of errors

Learners failed to identify properties of rectangles. It might be that they lack the basic knowledge of general geometry principles. Therefore, they find difficulty in applying the principles correctly. The largest proportion of learners (38,3%) chose option A (25); these learners may have resorted to answering the question with the information presented in the question itself.

### Recommended action

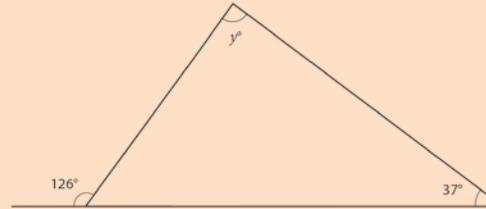
Classroom teaching should emphasise the properties of geometric figures, especially rectangles. Classroom exercises and testing should include complex examples from various resources.



Cognitive area: Applying

QUESTION 57

What is the value of  $y$ ?



M06\_10

Learner performance

	Correct	Incorrect	Omitted
South Africa	18,9	75,6	3,1
International average (n=38)	40,8	50,7	6,8

South African curriculum

Space and shape.

Types of errors

Learners continue to display lack of understanding of geometric techniques. Teachers might be unfamiliar with geometric concepts because of the phasing out in the OBE curriculum during the years 2008 to 2012.

Correct response was  $89^\circ$ . Learners must understand angles for both straight lines (must add up to  $180^\circ$ ) and triangles (must add up to  $180^\circ$ ). Although this can be calculated in a lengthier manner, learners can also use the knowledge that the angle on the straight line outside of the triangle adds up to the two opposite angles inside the triangle.

Incorrect responses were  $90^\circ$  or other incorrect. These learners did not understand the above and calculate accordingly. A common answer was  $17^\circ$  or  $163^\circ$  as indicated in the examples in Figure 6 and Figure 7. These learners either added 126 to 37 to get 163 or subtracted 163 from 180.

Figure 6: M06\_10 learner incorrect response example

$$y = 126 + 37 = 180$$

$$y = 183 - 163$$

$$y = 17$$

What is the value of  $y$ ?

Answer: 17

$y + 126 + 37 = 180$   
 $y = 180 - 163$   
 $y = 17$

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion



## Item-by-item analysis (continued)

**Figure 7:** M06\_10 learner incorrect response example

What is the value of  $y$ ?

Answer: 163

### Recommended action

Teacher training and support should include retaining of teaching of geometric concepts. The Department of Basic Education can assist by fast-tracking teacher support and re-teaching of geometry.

*Cognitive area: Applying*

**QUESTION 58** Draw the rest of the shape so that AB is the line of symmetry. ✖ 🗑 🖨

M07\_09

### Learner performance

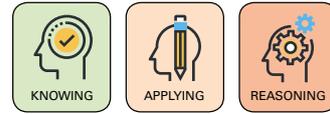
	Correct	Incorrect	Omitted
South Africa	23,5	47,6	28,0
International average (n=38)	50,9	30,6	17,6

### South African curriculum

Shapes and space: Symmetry.

### Types of errors

To answer this correctly, Point C must be within 2mm of correct intersection on the grid. South African learners confused symmetry and reflection. This displays a lack of knowledge of symmetry. A substantial proportion of learners (28%) did not even attempt this question.



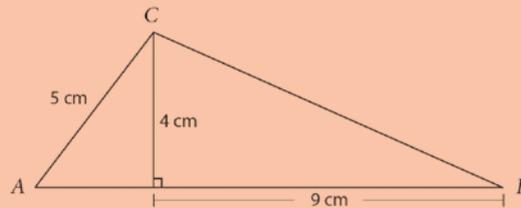
### Recommended action

Foundation phase teaching should emphasise symmetry. More emphasis needs to be placed on drawing the invisible line (drawing the line of symmetry). Intermediate phase teaching needs to create a solid foundation of symmetry that senior phase teachers can build upon. Teachers must use the CAPS curriculum document in conjunction with textbooks.

#### Cognitive area: Reasoning

#### QUESTION 59

What is the area of triangle ABC?



- A. 18 cm<sup>2</sup>
- B. 24 cm<sup>2</sup>
- C. 28 cm<sup>2</sup>
- D. 36 cm<sup>2</sup>

MO1\_12

### Learner performance

	A	B	C	D	Omitted
South Africa	64,8	11,4	7,3	13,0	2,0
International average (n=38)	30,1	30,6	18,2	16,5	3,3

### South African curriculum

Shapes and space: Pythagoras theorem.

### Types of errors

A majority of learners chose option A (18cm<sup>2</sup>); these learners have possibly resorted to adding the numbers presented in the question (9+4+5). Most learners focus on the given values and struggle to understand the various techniques required to employ the given values. In this particular instance, learners focused on the addition of available values. They failed to recognise the entire base of the triangle and instead focused on some of the components of the base that contained given values and angles. Learners at Grade 8 and 9 level struggle to identify complex procedures through reading.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

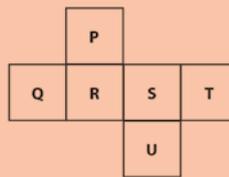
## Item-by-item analysis (continued)

### Recommended action

The inclusion of more information that would provide hints that the Pythagoras theorem must be applied would be helpful to the learners. Learners need to practise the appropriate techniques related to Pythagoras and angles through rigorous classroom exercises. Furthermore, they should be able to apply these techniques in word-problem sums. Teachers must differentiate in the type of questions they do in class, starting off with straightforward questions and working towards complex questions where they use skill to analyse the question.

### Cognitive area: Reasoning

#### QUESTION 60



Mariette constructed a cube by folding the net shown above.

Which square face is opposite face Q?

- A. P
- B. S
- C. T
- D. U

M02\_08

### Learner performance

Question 68	A	B	C	D	Omitted
South Africa	26,2	32,5	30,4	6,4	1,8
International average (n=38)	9,0	60,1	23,3	4,0	1,6

### South African curriculum

Space and shape: Geometry: 3-D objects.

### Types of errors

This question required learners to think abstractly about a 3-dimensional object. A majority of learners selected the two given faces closest to Q and not touching (options A and C). Learners did not demonstrate knowledge of properties of a cube.

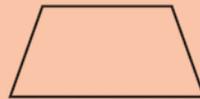
### Recommended action

The topic on 3-D nets is part of intermediate phase. Perhaps teachers should spend time building the 3-D objects from their nets in the earlier grades. Learners must touch and see the object to remember.

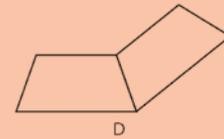
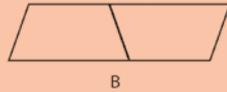
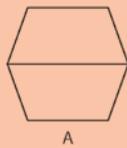


Cognitive area: Reasoning

QUESTION 61



Rami put two of these tiles together by matching the sides. He made four different shapes, as shown below.



Which of the two have the same shape?

- A. A and B
- B. C and D
- C. A and C
- D. B and D

M02\_09

Learner performance

Question 69	A	B	C	D	Omitted
South Africa	8,8	16,6	45,6	22,7	3,1
International average (n=38)	11,0	13,6	41,0	30,1	1,9

South African curriculum

Space and shape (Geometry).

Types of errors

Learners identified the figures which look almost the same (Figures A and C). Figure D is 3-dimensional; learners would have to be able to recognise this fact, recognise that the shapes would be the same in a 2 dimensional setting.

Recommended action

Teachers must explore using different techniques in calculating the perimeter, starting with pictures without known lengths and then increasing the level of difficulty through different values. They can also use examples that apply different combinations of 2-D figures.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

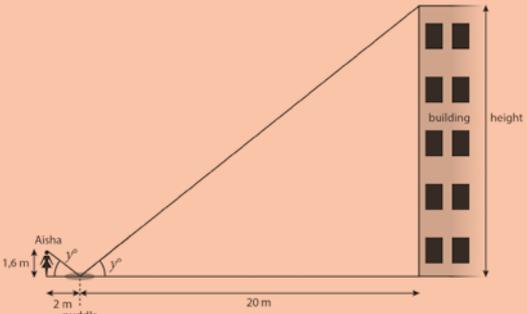
Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Reasoning**

**QUESTION 62**







Aisha is standing near a puddle in which she can see the reflection of the top of a building opposite. Her line of sight makes an angle of  $y^\circ$  with the puddle and reflections at the same angle.

Given the heights and distances shown in the picture, what is the height of the building?

M03\_10

### Learner performance

	Correct	Incorrect	Omitted
South Africa	3,5	90,3	5,4
International average (n=38)	15,1	68,1	16,2

### South African curriculum

Space and shape (Geometry) and Numbers.

### Types of errors

The correct answer is 16. Only 3,5% of South African learners answered this correctly. Learners could not identify the similar triangles and hence could not calculate the ratio between the given sides of the triangles. Learners struggle with the topics similarity and congruency in Grades 8 and 9.

### Recommended action

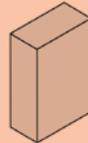
Learners should be encouraged to apply various maths concepts to solve different challenging questions. Teachers must constantly present problems that deepen cognitive thinking and ability.



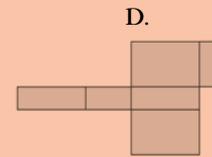
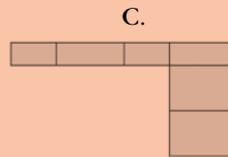
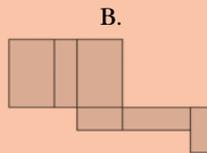
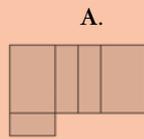
**Cognitive area: Reasoning**

**QUESTION 63**

The diagram shows a rectangle box.



Which figure can be folded to form the rectangular box?



M03\_11

**Learner performance**

	A	B	C	D	Omitted
South Africa	23,4	23,0	12,7	39,0	1,0
International average (n=38)	13,7	18,4	8,6	57,0	1,5

**South African curriculum**

Space and shape (Geometry): 3-D objects.

**Types of errors**

The high percentage of learners selecting A where there are only five faces is a big concern. Perhaps learners never see or touch or build any 3-D objects from Grade 4. The sizes of the different faces are not accurate in the drawings in B or D, raising concern as to how this influenced the learners' choice in B.

**Recommended action**

Teaching the basics from Grade 4 every year will improve the skill to match nets and objects. The learner must have the opportunity to build his or her own 3-D objects from nets or unfold a 3-D object to see the net. They must also understand there may be more than one net per object. The learners need to understand the importance of 3-D objects in normal life as packaging and other material, for example, shoe boxes, cool drink cans, sweets boxes, etc.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

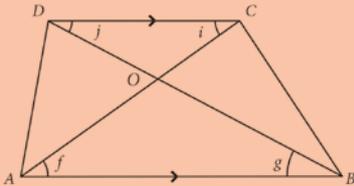
Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Reasoning**

**QUESTION 64**

In the figure, ABCD is a trapezoid. Triangles AOB and COD are similar.

Name the pair of angles that must be equal because these triangles are similar.

M05\_14

### Learner performance

	Correct	Incorrect	Omitted
South Africa	5,8	87,5	5,4
International average (n=38)	21,6	62,6	14,3

### South African curriculum

Space and shape (Geometry) – Quadrilaterals/Geometry of straight lines.

### Types of errors

Correct response: Either  $f = i$ , or  $g = j$ . The use of the word trapezoid vs trapezium may be a bit of a concern to the learners. It may depend on what textbook is used in class. Although the question refers to similarity, the equal angles are also equal due to other reasons. The question had no need to refer to similarity because  $j = g$  and  $i = f$  due to the parallel lines and alternate angles; even the third set of angles  $AOB = COD$  due to opposite angles. The learners struggle with similarity as stated in earlier questions and therefore they often fail to even try to answer questions including similarity.

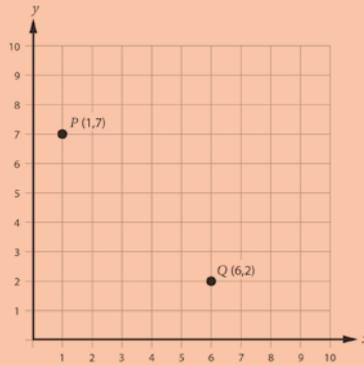
### Recommended action

The basics of line geometry start in Grade 7 where learners recognise similar 2-D figures and identify equal angles with parallel lines. The importance of constructions in Grade 7 must be recognised by the teacher as a method of teaching the properties around parallel lines. Learning the properties by heart is not enough for the learner to understand. Doing their own drawings in class and at home will improve their knowledge to remember what they have seen.



Cognitive area: Reasoning

QUESTION 65



The grid shows two points, P and Q. Which point is an equal distance from points P and Q?

- A. (7,8)
- B. (4,4)
- C. (3,5)
- D. (2,2)

M06\_11

Learner performance

	A	B	C	D	Omitted
South Africa	49,0	21,6	15,6	7,7	3,5
International average (n=38)	48,6	21,2	17,2	7,0	4,0

South African curriculum

Patterns, functions and algebra: Graphs and the plotting of points on the Cartesian plane.

Types of errors

The correct answer is A. This topic has been taught in the 4th term of Grade 8, hence the good results, even better than the international average. The high percentage of learners selecting B and C may be due to these two points lying somewhere between P and Q. Learners assume that to be the same distance from P and Q means the answer must be between P and Q.

Recommended action

Teachers need to understand that this basic drawing and identifying of points on the Cartesian plane is very important for the learners to develop in the senior mathematics phase in Grade 9 with the line graph, and in the FET phase with the new graphs. Learners must draw on paper from Grade 8 – start with pictures of animals or a house for example, make it a playing experience at first, and then moving to specific graphs. Learners need to move points around; this forms part of Space and shape (Transformations).

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

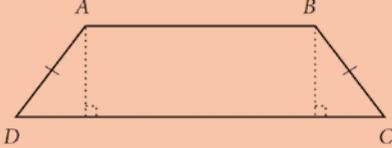
## Item-by-item analysis (continued)

**Cognitive area: Reasoning**

**QUESTION 66**







ABCD is a trapezoid with  $AB = 10\text{cm}$  and  $CD = 16\text{cm}$ .  $AD = BC$ .

The distance between the parallel lines  $AB$  and  $CD$  is  $4\text{cm}$ . What is its perimeter?

A. 36 cm  
B. 34 cm  
C. 32 cm  
D. 30 cm

M07\_10

### Learner performance

	A	B	C	D	Omitted
South Africa	19,7	19,4	15,1	42,6	2,0
International average (n=38)	31,8	30,2	14,8	18,7	3,7

### South African curriculum

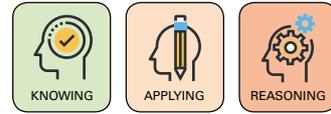
Measurement – Perimeter of 2-D figures.

### Types of errors

A very high percentage of learners simply added the available numbers – incorrect selection of D. A group of learners didn't understand the concept of "distance between 2 parallel lines". They used the value 4 twice as the length of  $AD$  and  $BC$ . Perhaps the word "perpendicular distance" would have helped in the wording although the perpendicular was shown in the figure.

### Recommended action

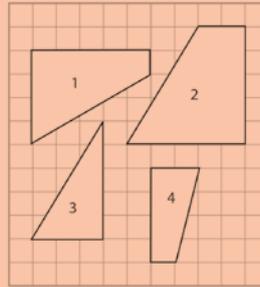
The Theorem of Pythagoras on its own is a challenge for most learners. To include this in a question with perimeter where they need to analyse the question on their own is even more challenging. It is very important that teachers show learners the integration of topics. Even starting off with sub-questions and leading the learners; ... first calculate the length of the side and in the next question calculate perimeter or area. Differentiate the type of questions when teaching is very important.



**Cognitive area: Reasoning**

**QUESTION 67**

Here are four shapes on grid.



Which two shapes fit together to make a square?

- A. 1 and 2
- B. 1 and 4
- C. 2 and 3
- D. 2 and 4

M07\_11

**Learner performance**

	A	B	C	D	Omitted
South Africa	33,5	20,6	26,4	16,4	1,7
International average (n=38)	26,2	23,7	39,0	8,5	1,7

**South African curriculum**

Space and shape (2-D figures – identifying).

**Types of errors**

Learners selected figures that look alike. Perhaps they did not read the question thoroughly or failed to understand the concept of building a new figure from different 2-D figures. Their lack of knowledge on the properties of a square may also be a factor.

**Recommended action**

Learners start with the properties of rectangles and squares in Grade 4 and every year they include new quadrilaterals. Perhaps teachers are not teaching this part of geometry or learners fail to understand the building of figures due to lack of the opportunity to build them in the classroom with the teacher. Building figures in class from Grade 4 to 6 is important to apply the properties in Grades 7 to 9. Teachers must help learners to cut different 2-D figures and show them how to build a new figure in different ways – practical learning (do – see – remember – know).

Contextual factors and icons  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

### TIMSS content area: Data and chance

*Cognitive area: Knowing*

**QUESTION 68**  Thembi surveyed 15 girls and 15 boys about the number of hours per day they use electronic devices.  

	Number of hours spent using electronic devices per day	Total number of hours
Boys	2,2,2,3,3,3,3,3,3,3,4,4,4,4,5	50
Girls	0,1,1,2,2,2,2,2,2,3,3,3,4,4,5	36

Which statement about this data is true?

- The mean of the girls' data is greater than the mean of the boys' data.
- The median of the girls' data is greater than the median of the boys' data.
- The mode of the boys' data is greater than the mode of the girls' data.
- The range of the boys' data is greater than the range of the girls' data.

M02\_11

#### Learner performance

	A	B	C	D	Omitted
South Africa	13,0	12,6	38,7	27,0	3,9
International average (n=38)	8,4	10,5	51,0	22,9	3,9

#### South African curriculum

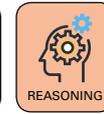
Data handling.

#### Types of errors

The meaning of mode and median starts in Grade 6 and from Grade 7 the mean and range is taught. Learners are confused because of the length of the question – reading disability or not understanding what they are reading. The high percentage of learners selecting option D shows that learners lack basic knowledge – perhaps believing that “total number of hours” refers to range.

#### Recommended action

Starting with this data-handling topic in the third term of Grade 4 there must be progress in teaching the content. Revise the previous year's study and add new knowledge. Learners must practise different types of questions on this topic. In this type of question, where they have to analyse the data on their own to understand each distractor, the learner might be confused if the teacher only concentrates on single types of questions. Don't always ask questions such as: Calculate the mean, Write down the mode; also explore with higher cognitive type of questions.



**Cognitive area: Knowing**

**QUESTION 69**



A machine has 7 different colours of gumballs. Liv noticed that people bought 306 gumballs and that 23 of them were blue.



Which is the best statement that the next gumball will be blue?

- A.  $\frac{1}{7}$
- B.  $\frac{7}{23}$
- C.  $\frac{7}{306}$
- D.  $\frac{23}{23}$

M02\_12

**Learner performance**

	A	B	C	D	Omitted
South Africa	8,9	22,7	19,5	42,1	1,6
International average (n=38)	14,5	17,4	15,8	46,0	2,3

**South African curriculum**

Data handling: Probability.

**Types of errors**

Learners did not understand that the concept of probability relies on the total number of outcomes, therefore they select B as an option. The word “estimate” versus “probability” may be of concern for some learners. Perhaps learners failed to recognise that the question refers to “blue” only. Language may be an issue here. Due to slow coverage of topics through the year, these final concepts in CAPS are often ignored by the teacher.

**Recommended action**

Teachers must recognise the importance of basic probability knowledge from Grade 4 because this topic runs through to Grade 12. Although they revise every year, there must be an increase in the level and type of questions for the learner to progress in this topic. Because this topic often ends up in the 4th term, teachers must spend the given time on the topic. Teachers must keep track of the timeframe in CAPS and adhere to it.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 70**



Matselane has a bag of 24 marbles. The bag has 8 blue marbles, 8 red marbles and 8 white marbles. Matselane picks 1 marble at random out of the bag.

What is the chance that the marble is blue?

A.  $\frac{1}{3}$       B.  $\frac{1}{8}$

C.  $\frac{3}{8}$       D.  $\frac{1}{24}$

MD2\_13



### Learner performance

	A	B	C	D	Omitted
South Africa	13,7	23,1	17,4	38,6	1,2
International average (n=38)	39,2	18,1	13,0	23,6	1,3

### South African curriculum

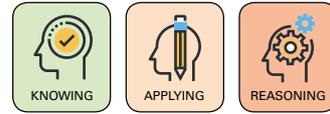
Data handling: Probability.

### Types of errors

Learners have no idea of the concept of probability where more than one colour of a ball applies. Learners just selected numbers from the question randomly. Having only one distractor with the total number of balls (24) made the majority of learners select option D. Options B and C were selected because the learner recognised the eight blue balls in the question. Distractor A was in simplified form and therefore learners failed to recognise it as a possibility.

### Recommended action

Include questions in your exercises and common tasks where learners do have to simplify probability questions.



**Cognitive area: Knowing**

**QUESTION 71**

**Morning Temperature in Zed City**

The graph shows hourly temperatures from 7am and 11am. Estimate the temperature at 9:30am.

M05\_16

### Learner performance

	Correct	Incorrect	Omitted
South Africa	21,8	73,3	2,7
International average (n=38)	52,0	40,0	5,6

### South African curriculum

Data handling: Interpreting of data.

### Types of errors

Correct response: 20,4 – 20,7 (inclusive). Learners are unable to read the values on the two axes because there are no markings halfway between the given values on the labels. Learners fail to understand the meaning of “estimate”. It is not clear, should the question be rephrased, if the learners would understand better how to use the given graph (e.g., use the graph and write down the approximate temperature at 9:30am)

### Recommended action

Interpretation of graphs starts in Grade 4. Teachers need to make sure learners practise different questions regarding the interpretation of graphs. Questions must be on different levels too, including not only knowledge. Learners can even use different graphs from newspapers.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

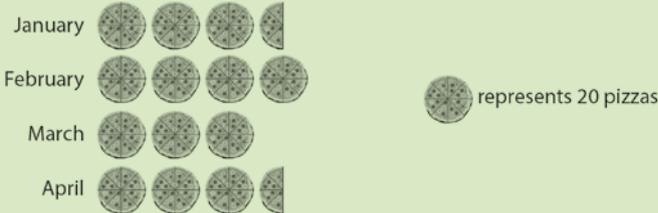
Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 72** 

The pictogram shows how many pizzas a shop sold in four months. 



One of these tables shows the same information, which one?

A. 

Month	Pizza
January	60
February	80
March	60
April	60

B. 

Month	Pizza
January	70
February	80
March	60
April	70

C. 

Month	Pizza
January	70
February	140
March	60
April	70

D. 

Month	Pizza
January	60
February	80
March	70
April	60

M06\_13

### Learner performance

	A	B	C	D	Omitted
South Africa	14,3	56,8	14,5	7,6	2,2
International average (n=38)	7,2	78,0	6,6	3,2	1,5

### South African curriculum

Data handling: Interpretation of graphs.

### Types of errors

Learners are unable to understand the use of a picture to illustrate data. While more than 50% do know that a pizza can be sliced in half, 21% of learners ignored the last picture in the pictographs. Pictographs are not part of Grade 7 to 9 CAPS and therefore learners don't realise they must remember it from their previous years.

### Recommended action

In Grade 4 the learners start with pictographs as their very first attempt to illustrate data. Teachers need to give the learners enough exercises up to Grade 6 in reading or interpreting pictographs.



**Cognitive area: Knowing**

**QUESTION 73**

This picture shows the speedometer of a car.



What speed is shown on the speedometer?

- A. 77 km/h
- B. 82 km/h
- C. 84 km/h
- D. 86 km/h

M07\_02

**Learner performance**

	A	B	C	D	Omitted
South Africa	35,7	25,4	26,1	12,1	0,5
International average (n=38)	18,4	20,1	54,9	5,7	0,7

**South African curriculum**

Data handling: Interpretation.

**Types of errors**

Most learners assumed that every small line counts as one. Perhaps the use of a speedometer was a strange concept because not all learners know how a speedometer works or have ever seen one in a car or anywhere else. The understanding of scale units is important as pre-knowledge to this question. If learners understood how to read a clock in previous years, this should have been easy. Other learners could identify the longer line halfway between 70 and 90km/h as 80km/h but then counted every small line as one unit. Learners often struggle to identify detail in a question.

**Recommended action**

Classroom exercises should include questions in exercises and tasks where learners have to sift through a problem statement, identify the details and key information and construct responses using the key information.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

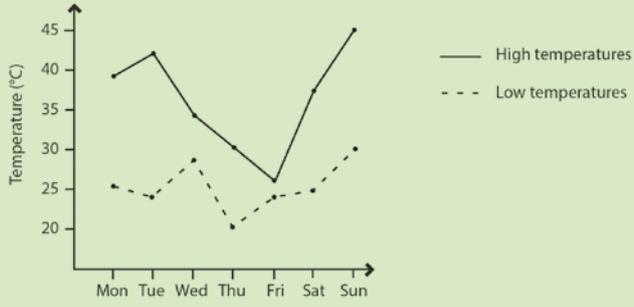
## Item-by-item analysis (continued)

**Cognitive area: Knowing**

**QUESTION 74**



**Temperature Chart of a Week in Zedland**



The graph above shows the daily high and low temperatures for a week at a place in Zedland. On which day was the difference between the highest and lowest temperatures 10 °C?

A. Wednesday  
B. Thursday  
C. Friday  
D. Saturday

M07\_12

### Learner performance

	A	B	C	D	Omitted
South Africa	9,6	52,5	20,0	13,9	2,4
International average (n=38)	11,5	61,8	12,3	11,2	2,0

### South African curriculum

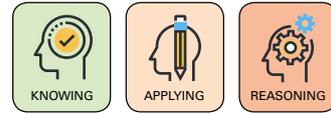
Data handling: Interpret data.

### Types of errors

The 20% of learners selecting C as an option shows that perhaps learners struggle to read the approximate values from the graph. The data is somewhere between the given labelled values. The learners may have guessed incorrectly. As stated before, many learners struggle to understand and read scale values on a graph.

### Recommended action

The broken-line graph is the only addition to interpretation of graphs in Grade 8 in the third term. Teachers need to emphasise the new work and make sure learners do enough exercises when they start data in Grade 8. Learners must understand the use of different scales before starting with the graphs. Revision of prior knowledge is important.



**Cognitive area: Knowing**

**QUESTION 75**



The weather forecast for tomorrow in Zedland is below.



30% chance of rain

How likely is it that it will rain tomorrow in Zedland?

- A. It is certain to rain.
- B. It is likely to rain.
- C. It is unlikely to rain.
- D. It will not rain.

M07\_14

**Learner performance**

	A	B	C	D	Omitted
South Africa	15,2	35,1	30,5	15,3	1,1
International average (n=38)	4,1	29,5	58,6	5,0	1,0

**South African curriculum**

Data handling: Probability.

**Types of errors**

Learners don't know the terminology regarding the probability scale (0% = "will not" and 100% = "it will" and the rest in between. Learners could even misunderstand the information given in the picture with the two clouds meaning it is likely to rain, and as a result not really concentrate on the information next to the picture.

**Recommended action**

Starting in Grade 7 learners need to know certain words are equivalent to certain percentages and equivalent to a certain probability.

Contextual factors and items

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 76**

The heights of 100 students in a school were measured to the nearest 5 cm.

This table shows the results.

Height (cm)	145	150	155	160
Number	16	40	25	19

Complete this bar chart to show the same information.

**Students' Heights**

M01\_13

### Learner performance

	Correct	Incorrect	Omitted
South Africa	52,3	38,7	7,2
International average (n=38)	63,6	27,1	7,3

### South African curriculum

Data handling: Represent data.

### Types of errors

Correct response: Bar for 155 between 24 and 26 (exclusive), bar for 160 between 17,5 and 20 (exclusive), and no other bars drawn. A high percentage of learners can draw bar graphs. The few struggling may be due to interpretation of the given scales as discussed in earlier questions.

### Recommended action

Learners must complete enough exercises in drawing and completing bar graphs with given scales and exercises where they select their own scales to draw. Repeating these different questions from Grade 7 makes it easier in Grades 8 and 9. Learners also use bar graphs in other subjects, for example geography and natural sciences.



### Cognitive area: Applying

#### QUESTION 77



Television Programmes Liked by Students



The graph shows the type of television programmes 240 students say they like most.

Which one of these is the number of students that chose History?

- A. 20
- B. 30
- C. 40
- D. 60

M03\_14

#### Learner performance

	A	B	C	D	Omitted
South Africa	34,7	35,1	14,8	12,5	0,9
International average (n=38)	22,4	51,9	16,4	5,6	1,8

#### South African curriculum

Data handling: Interpret data.

#### Types of errors

Learners could identify that one sector is equal to 20 but were unable to interpret half of the sector. Therefore they selected the answer for one sector. Again the learners struggle with interpretation of scale. This question is equivalent to understanding a clock or watch.

#### Recommended action

Teachers must make learners aware of the different types of pie charts in the real world too, for example, watches (clock), pizza, etc. Exercises with different scales and meaning must be practised from Grade 7 on a regular basis. Never assume learners have done the topic thoroughly in the previous year.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Applying**

**QUESTION 78**

A company makes cell phones. The table below shows the types of phone they make and the percentage of sales for each one.

Type of Cell Phone	Percentage of Sales
Supertext	40%
Bigview	20%
Economy	25%
Luxury	15%

Which pie chart shows this information correctly?

A. B. C. D.

M05\_15

### Learner performance

	A	B	C	D	Omitted
South Africa	12,5	42,2	32,9	7,9	2,7
International average (n=38)	12,1	62,7	13,0	8,3	1,9

### South African curriculum

Data handling: Represent data.

### Types of errors

Learners who selected option C had no idea of scale, where 40% is less than half the pie chart. They just selected a pie chart randomly as the correct one. Perhaps learners struggled because they have to guess the given percentages to a certain sector. If the degrees were given in the pie chart then it would be a fair question. Learners could then have calculated the correct degrees from the given percentages.

### Recommended action

Learners must be taught how to divide a pie chart into sectors of different degrees and how to convert percentages to degrees or fractions and the other way around. They can work on rough sketches to make comparisons.

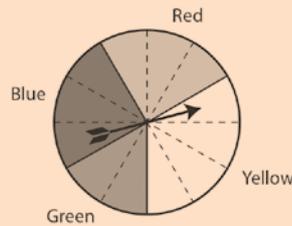


Cognitive area: Applying

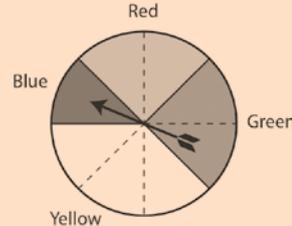
QUESTION 79



Mbali's Spinner



Ben's Spinner



Suppose Mbali and Ben spin their spinners.

A. Which of the following is true?

- A. Blue is more likely on Ben's spinner than on Mbali's
- B. Blue is more likely on Mbali's spinner than on Ben's
- C. Blue is impossible on both spinners
- D. Blue is equally likely on both spinners B.

B. Which of the following is true?

- A. Red is more likely on Ben's spinner than on Mbali's
- B. Red is more likely on Mbali's spinner than on Ben's
- C. Red is impossible on both spinners
- D. Red is equally likely on both spinners

M06\_12

Learner performance

Part A	A	B	C	D	Omitted
South Africa	23,9	54,9	8,9	7,3	1,4
International average (n=38)	16,8	65,7	5,9	6,4	2,6

Part B	A	B	C	D	Omitted
South Africa	17,2	32,1	15,1	29,6	1,6
International average (n=38)	13,9	19,6	8,9	51,7	2,7

South African curriculum

Data handling: Probability.

Types of errors

For part A, the learners who selected option A (23,9%) looked at the direction of the arrow and not the size of the sector.

For part B, learners may have counted the number of sectors and ignored the different sizes of the two given graphs. Perhaps learners cannot convert the eight sectors to 45° each and the 12 sectors to 30° each. Learners couldn't link the sector size to determine probability.

Contextual factors and icons

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion

## Item-by-item analysis (continued)

### Recommended action

When learners start with spinners in Grade 7 they must explore different exercises and types of questions on the outcomes of a spinner. Teachers should emphasise the consideration of the size of the sector to determine probability.

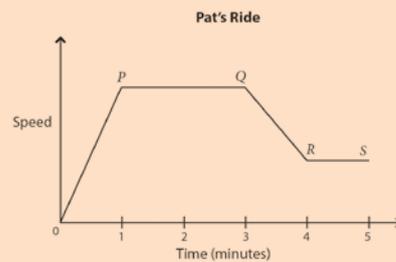
From Grade 7 learners need to know that the sector size can change from spinner to spinner. Teachers must use different spinners and ask different questions to test and improve terminology. This can be done by emphasising the importance of considering the size of sectors to determine probability.

### Cognitive area: Applying

#### QUESTION 80



Pat set out for a bicycle ride. The graph shows her speed over the first 5 minutes of her ride.



Which is a possible explanation for the part of the graph between Q and R?

- A. Pat was cycling uphill.
- B. Pat was cycling downhill.
- C. Pat stopped for a minute.
- D. Pat cycled back towards the start.

M07\_13

### Learner performance

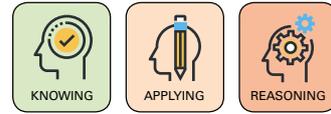
	A	B	C	D	Omitted
South Africa	23,5	41,8	20,7	9,8	2,0
International average (n=38)	40,9	38,0	12,2	5,5	1,8

### South African curriculum

Data handling: Interpret graphs.

### Types of errors

Learners are unsure of the slope of a graph and the interpretation of the slope. The drawing of line graphs is covered in Grade 9 and therefore learners struggled. Selecting option B, where the graph itself goes downwards, the learners selected a downhill option as answer.



### Recommended action

The learners will have to do more exercises in Grade 9 to improve their skill level in interpretation. The topic Algebra: Graphs in Grade 9 will improve their thinking around slopes (gradients). Learners need this knowledge in physical science too.

**Cognitive area: Reasoning**

**QUESTION 81**




**Books Sold**

Number of Books

Months

A salesman looked at the graph showing his sales of books for the first 6 months of 2004, and said, "In March, I sold four times as many books as I sold in February."

Explain whether you agree or disagree with the salesman, and give a reason.

M01\_14

### Learner performance

	Correct	Partially correct	Incorrect	Omitted
South Africa	0,3	9,9	79,5	7,6
International average (n=38)	2,5	32,5	47,7	12,6

### South African curriculum

Data handling: Analyse data.

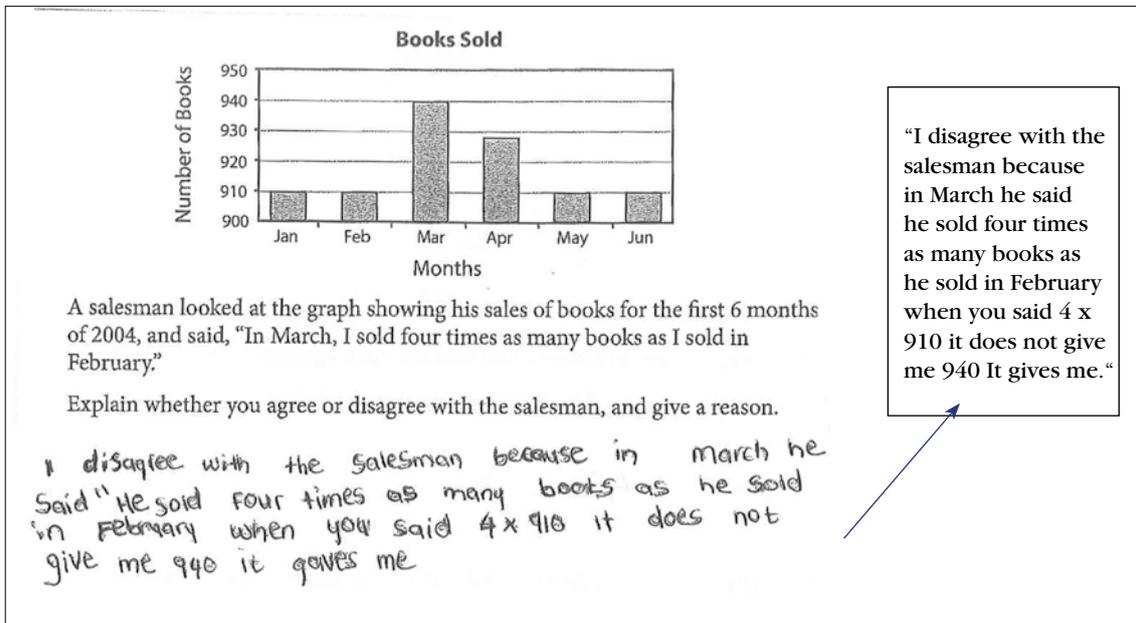
### Types of errors

Correct responses stated that the learner disagreed with reference to a false origin or the scale not starting from zero. Correct responses stated that the learner disagreed with an explanation based on multiplication or division. An example of an explanation stated: "I disagree because in March he sold three times the books he sold in February." Correct responses stated that the learner disagreed with an explanation that the increase cannot be four times as many books. An example is provided in Figure 8.

Contextual factors and items  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

Figure 8: M01\_14 learner correct response example

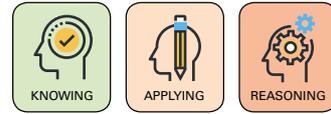


Incorrect responses could be either agree or disagree with an explanation based on only relative heights of the bars shown. An example stated: "I disagree because in March he sold 950 books and in February he sold 910 books."

Incorrect responses were all other answers given. One answer stated: "I agree because on the graph I shows that on March he sold more books" which shows that the learner is not able to recognise that bars on the provided graph cannot be compared as a ratio (i.e. the graph does not begin from zero). "I disagree because in 2000 they were not sales of books" was another answer provided which, together with other fabricated answers, clearly indicates confusion. Some learners did recognise the scale on the y-axis and could identify that March sales are only 30 more than February. Other learners didn't check the scale but looked at the lines on the graph and stated that the four lines in March are four times the one line in February.

### Recommended action

Learners must spend time evaluating given scales on a graph and critically discuss it in classroom, led by the teacher asking different questions. Practising this from Grade 7 will improve the learners' skill in analysing information depicted in graphs.



**Cognitive area: Reasoning**

**QUESTION 82**



Ahmed had the following scores out of 10 on his first 4 mathematics tests: 9, 7, 8, 8. Ahmed has 1 more test with a maximum of 10 points and says he wants to get an overall average of 9. Is it possible for him to do this? Explain your answer.



M01\_15

**Learner performance**

	Correct	Incorrect	Omitted
South Africa	2,6	79,1	11,2
International average (n=38)	24,9	52,7	12,9

**South African curriculum**

Data handling: Report data.

**Types of errors**

Correct response: No, with adequate justification, e.g., he would have to score 13/he can only average 8,4/he needs 45 points but can only get 42/or equivalent. Over 79% of learners did not answer this question correctly. Learners may have calculated the average of the four tests as 8 marks and assumed that Ahmed will not end with an average of 9. Others calculated the average as 8 marks and then calculated the average of 8 and 10. They will state that he can end with an average of 9, but the explanation is incorrect.

**Recommended action**

Learners must start in Grade 8 with predictions based on the mean score. Teachers must be aware that questions can be asked in different ways and teach learners the necessary methods. This reasoning question is not just on calculating the mean as a routine question but a little more complex; questions with a twist.

Contextual factors and icons  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

## Item-by-item analysis (continued)

**Cognitive area: Reasoning**

**QUESTION 83**

**School Sports Survey – Grades 7–10**

Percent of students who chose soccer as their favourite sport:

Grade	Percent of students
Grade 7	75%
Grade 8	65%
Grade 9	72%
Grade 10	70%

Tebogo’s school asked students in Grades 7–10 to choose their favourite sport. There are 100 students in each grade. The graph shows the results for the students choosing soccer. Tebogo compared the results for Grade 7 and Grade 8. He thought twice as many students in Grade 7 as in Grade 8 chose soccer.

Explain how the graph led Tebogo to make this mistake.

M03\_13

### Learner performance

	Correct	Incorrect	Omitted
South Africa	7,5	71,4	20,0
International average (n=38)	27,2	44,1	27,5

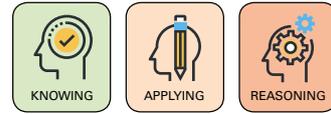
### South African curriculum

Data handling: Analyse data.

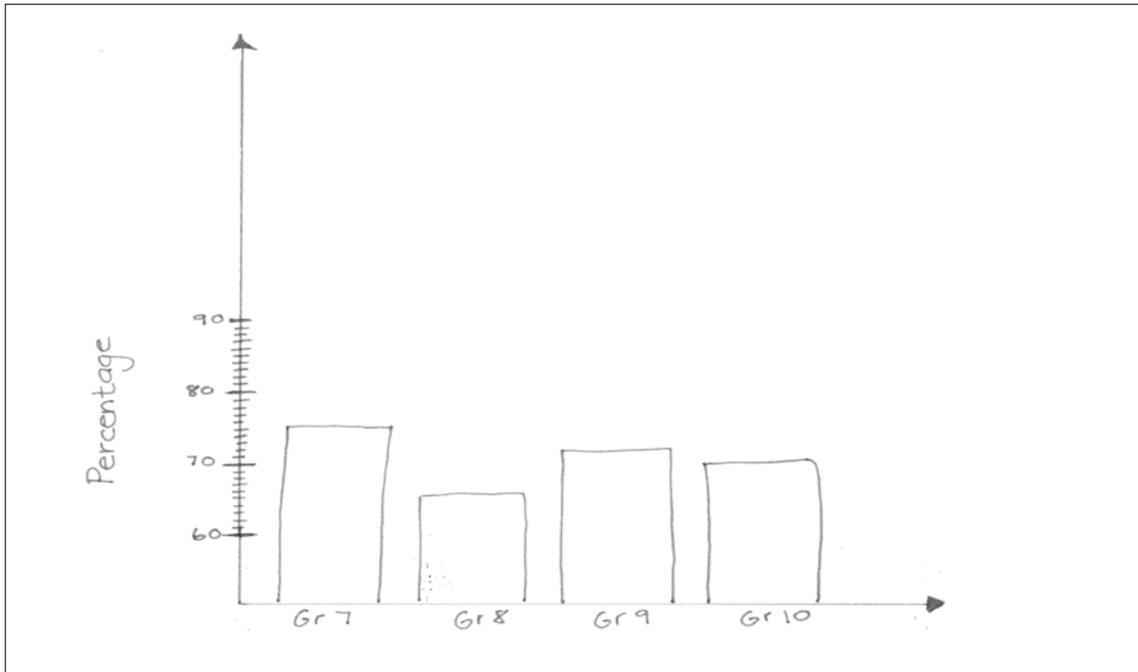
### Types of errors

Correct answers stated that the Grade 7 bar is twice as long as Grade 8 bar or equivalent OR that the origin is not at 0 OR that the graph is not drawn to scale. Incorrect responses included all other answers provided. There were no common error trends but the majority of learners were incorrect.

One learner, as indicated in the excerpt below, drew a different bar graph rather than provide an explanation. Learners also became confused as to what they should have critiqued. For example, some learners were critical of Tebogo’s data-collection and analysis methods rather than the graph. Some learners restated the problem, i.e. that Tebogo had made a mistake. This suggests that South African learners struggle to express themselves in words and/or become confused at what is expected of them. See the example in Figure 9.



**Figure 9:** M05\_13 learner incorrect response example



Learners have to look at the length of the block and the given percentages. Learners using visual awareness will immediately draw the incorrect conclusion because the Grade 7 bar is twice the length of the Grade 8 bar. Learners who understand that the use of the given percentages with each bar makes the size of the bar misleading, will recognise that the conclusion is incorrect.

### Recommended action

When teachers teach graphs and analysing graphs they need to teach learners to focus on the type of graph and the scale together. Even in their selection of a scale, when they draw graphs, it is important to ask “what must my graph tell me?”. A wrong scale can lead to a wrong interpretation.

Contextual factors and icons  
Overall weaknesses identified  
Overall strengths identified  
Item-by-item analysis  
Remedial actions  
Conclusion

# 5. Remedial actions

## Factor one: Language



Teachers must motivate learners in the use of comprehension skills in class when complex language and terminology are used. There needs to be a gradual progression from simplified language or problem statements in the foundation phase to complex problem statements in the senior phase.

## Factor two: Back to basics



The big learning curve starts in Grade 4 where learners are confronted with too much content in CAPS and not enough teaching and consolidation time. This places pressure on teachers to cover the content on a very basic level. When learners progress to the senior phases they battle with remembering knowledge from the foundation phase. Therefore, it is necessary for foundation phase teachers to balance the time and content constraints without compromising the knowledge of learners.

## Factor three: Curriculum



As mentioned for previous factors, teachers struggle with getting through the CAPS curriculum while simultaneously ensuring that learners have a solid grasp of foundation phase mathematical concepts and principles. The development of a concise curriculum and textbooks that incorporate the basic mathematical principles and comprehensive examples and exercises would be helpful in alleviating these challenges for teachers and learners.



## Factor four: Teacher training and support



The South African curriculum has gone through some changes over the past years. The changing curriculum hasn't been accompanied by consistent teacher training and support. The reintroduction of geometry into the curriculum necessitates this training and continuous support. The continuous support can also be accompanied by supplemental material with rigorous classroom exercises. This can be used to complement the CAPS textbook which often lacks adequate examples. Furthermore, teachers need more training in the setting and assessment of assignments, projects, tests, exams and investigations.

## Factor five: Exam and testing techniques



The retraining of teachers can include exam and testing techniques that can be transferred to learners. During this transfer teachers must devise different ways of testing the same topic using progressed levels. The difference between the levels in CAPS must be simplified and progress towards complex examples to make it manageable and understandable for both the teacher and the learner.





## 6. Conclusion

Central to the learner performance in assessments (both national and international) in mathematics is the ability to solve problems. The National Curriculum Statement Grades R-12 (DBE, 2011) specifies that it aims to produce learners that are able to identify and solve problems and make decisions using critical and creative thinking. The National Council of Teachers of Mathematics (NCTM) has strongly endorsed and to a greater extent encouraged the inclusion of problem solving in school mathematics (1980, 1989). Jitendra and Star (2011) argue that one of the reasons why the learners are struggling with problem solving and therefore lag behind those who perform well is that problem solving is not well addressed in many mathematics textbooks. This argument can be taken even further: that there are insufficient problem-solving activities in mathematics classrooms. As a result, learners find it very difficult when they encounter problem-solving activities in various forms and levels of assessment. Giganti (2007) argues that there are three parts to learning mathematics: skills, concepts, and solving problems. Skills are the tools of mathematics, such as the ability to add and manipulate numbers. Concepts are the ideas in mathematics, such as the concept of a triangle. Problem solving is the ability to apply mathematics in different situations.

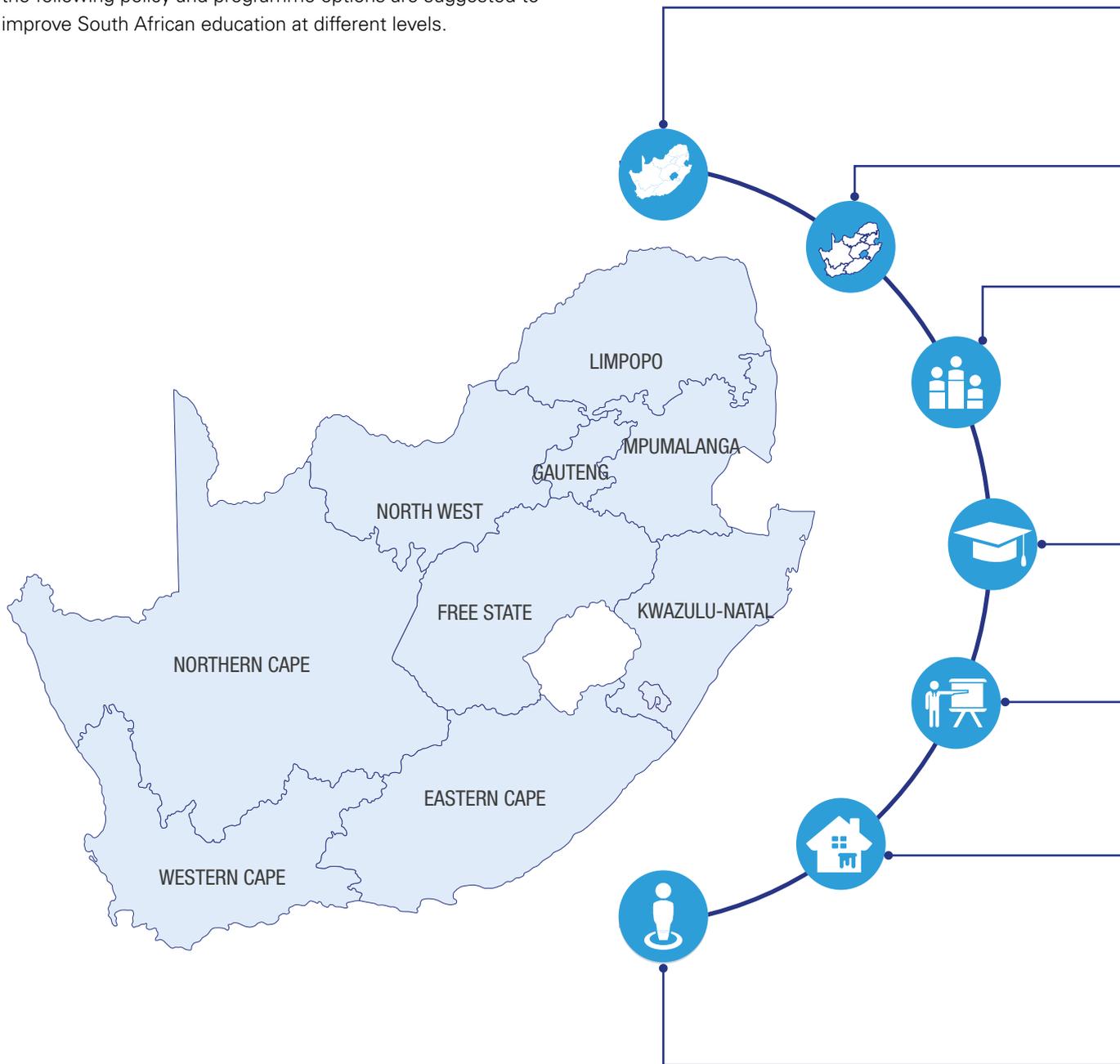
The analysis of the kinds of errors that learners made in TIMSS 2015 suggests that the area of problem solving still needs considerable attention in the mathematics classroom. The examples that are included in this item-by-item analysis give an idea about the ability of the learners to solve problems. It is critical that problem-solving skills are inculcated on a regular basis in the mathematics classrooms. Hopefully this may translate to improved abilities of the learners in solving problems and in the process, improve on performance.

This item-by-item analysis also provides the teachers with an indication of the performance of learners in different content areas. It can therefore be used to address the areas in which the learners have not performed well and in which teachers would be required to devise and develop appropriate strategies to improve the performance.



## How do we improve the chances of success for South African learners?

Based on the analysis of the TIMSS 2015 data for South Africa, the following policy and programme options are suggested to improve South African education at different levels.





## National level

It is important for the national government to:

- Ensure that the delivery of mathematics and science courses in all schools is in line with curriculum requirements
- Work towards an improvement in performance in national and international assessments based on realistic targets
- Embark on a differentiated strategy of interventions and support for improving learning outcomes in fee-paying and no-fee schools
- Improve the provision of pedagogical infrastructure and resources to schools
- Use public media to create awareness and set up programmes in mathematics and science
- Revisit the repetition policy and its implementation
- Generate awareness of the full range of career possibilities that learners can pursue beyond Grade 9
- Develop policies and guidelines to curb school violence and bullying

## Provincial level

It is important for each provincial department of education to:

- Increase the percentage of learners achieving above the minimum competency level of 400 points
- Monitor the provision and use of pedagogical resources in schools
- Emphasise a high quality of teaching and learning from Grade R
- Implement policies and guidelines to curb school violence and bullying

## District level

It is important for district education officials to:

- Design appropriate pedagogical interventions for teachers
- Monitor that teachers and learners are in school, on time and teaching/learning
- Monitor that textbooks, workbooks and pedagogical resources are in schools and being used
- Investigate teacher job satisfaction and motivation
- Monitor incidences of violence and bullying at schools, and support the management of school safety

## Schools

It is important for school management teams to:

- Emphasise safety, order and academic success
- Monitor and manage rates of absenteeism among teachers and learners
- Emphasise an academic culture in schools;
- Provide appropriate support to grade repeaters, either during school time or during school holidays
- Implement policies and guidelines to curb school violence and bullying

## Teachers

It is important for teachers to:

- Ensure that they arrive at school on time to start teaching
- Evaluate their own professional knowledge and pedagogical practices, and improve on these
- Provide learners with practice examples involving written explanations

## Communities and households

It is important for community members and families to:

- Motivate young children to see the importance of education
- Motivate and inspire young children to value mathematics and science
- Monitor teacher and learner attendance at schools
- Support and monitor homework and school reports
- Engage with teachers and school officials about education delivery and performance

## Learners

It is important for learners to:

- Arrive at school, and to their classes, on time
- Improve their proficiency in the test language
- Ensure regular practice of mathematics and science examples with written homework

Contextual factors and issues

Overall weaknesses identified

Overall strengths identified

Item-by-item analysis

Remedial actions

Conclusion



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Mathematics  
Fractions  
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Knowing Pattern  
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